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Galois cohomology and birational invariant of algebraic varieties

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Introduction.

This paper was written motivated by Manin's recent paper [**4**] in which he suggested the importance of Tamagawa number of the dual torus of the Neron-Severi group of a rational surface, in connection with the ζ-function of the surface. But in this paper we shall deal with arbitrary dimensional algebraic varieties without the restriction of the rationality and define some birational invariant of them. When we consider only the rational varieties, we can define the birational invariant using only the Neron-Severi groups of them but for arbitrary algebraic varieties we must take into account the contributions of the Albanese varieties of them.

Since we use the arguments developed in T. Ono's paper [**6**], we have to restrict the basic field k to a field of dimension one.

Let k be a field of dimension one i. e. either a finite algebraic number field or an algebraic function field of one variable over a finite field. Let V be a complete non-singular algebraic variety defined over k. Let $N^0(V)$ be the torsion free part of the Neron-Severi group of V (i. e. $N^0(V)=D(V)/D_t(V)$; $D(V)$ is the group of all divisors on V and D_t is the group of torsion divisors), and A be the Albanese variety of V defined over k. Let $\mathrm{Hom}(A, \hat{A})$ be the finite type Z-free module of all the rational homomorphisms from A to $\hat{A}$. Then the birational invariant $\mu_k(V)$ of V over k will be defined by

$$\mu_k(V)=\boldsymbol{h}_k(V)/\boldsymbol{i}_k(V),$$

$$\boldsymbol{h}_k=\frac{h_k^1(N^0(V))}{h_k^1(\mathrm{Hom}(A, \hat{A}))^{1/2}}, \qquad \boldsymbol{i}_k(V)=\frac{i_k(N^0(V))}{i_k(\mathrm{Hom}(A, \hat{A}))^{1/2}}$$

where h_k^1 and i_k are the notations used in [**5**] (see § 3). When V is a rational variety, the Albanese variety A vanishes and $\mu_k(V)$ depends only on $N^0(V)$. In § 2 and § 3 we show that $h_k^1(N^0(V))$ and $i_k(N^0(V))$ are birational invariant over k. (See Theorem 2 and Proposition 6). Since the Albanese variety A attaches to V birational-invariantly, we see that $\mu_k(V)$ is a birational invariant of V over k. The reason why we have considered the contributions of the