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Geometrical operations of Whitehead groups

By Mitsuyoshi KATO*

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§ 1. Introduction.

In the paper, we study PL manifolds which are related by h-cobordisms.

Suppose that we are given a PL manifold M of dimension $n \geqq 5$ with $\pi_1(M) = G$. An element τ of $\mathcal{W}h(G)$ operates on M in such a way that the result $M \cdot \tau$ of the operation is the right end of an h-cobordism U from the left end M with $\tau(U, M) = \tau$. This operation is called an interior operation. A result of Milnor ([15], Theorem 11.5) was concerned with the inertia group of this interior operation. If M is located on the boundary of a PL manifold W of dimension $n+1$, then we obtain a new PL manifold pair $(W \cup U, M \circ \tau)$, of which we may think as to be obtained from (W, M) by an operation of τ. This operation is called a boundary operation. A study of the boundary operation gives us a rough information about PL homeomorphism classes of compact PL manifolds whose interiors are PL homeomorphic.

In order to make rigorous definitions of these operations (especially the boundary operation) we need Corollary 2.3 which is deduced from the existence and uniqueness Theorem of embedded h-cobordisms (Theorems 2.1 and 2.2). These are slight modifications of results for abstract h-cobordisms due to Stallings ([18], p. 250) and Milnor ([15], Theorem 11.3) and may be well-known.

In § 3, we give the precise definition of the interior and boundary operations and obtain an extension of Milnor's result for boundary operations, see Theorem 3.4. In particular, for a compact PL manifold W of dimension $n = \text{odd} \geqq 7$, it is proved that there are finitely many distinct PL homeomorphism classes (respectively h-cobordism classes) of compact PL manifolds whose interiors are PL homeomorphic to $Int\ W$, provided that $\pi_1(bW)$ is finite and that $\pi_1(W) = 1$ (resp. $\pi_1(bW) \cong \pi_1(W)$), see Corollary 3.5.

Suppose that we are given an abstract regular neighborhood N of a polyhedron P. For each element τ of $\mathcal{W}h(\pi_1(bN))$, we have a new neighborhood $N \cup U$ of P as the result of a boundary operation of τ on (N, bN), where U

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