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On the asymptotic behaviour of the Green operators for elliptic boundary problems and the pure imaginary powers of some second order operators

By Daisuke FUJIWARA

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§ 0. Introduction.

In this note we shall generalize the results of the author's previous papers [7] and [8] to the case of general elliptic boundary problems of even order.

Suppose X and Y are respectively smooth vector bundles over a compact oriented Riemannian manifold M and its boundary ∂M. Let A be an elliptic partial differential operator operating on smooth sections of X and let B be a boundary differential operator mapping sections of X to those of Y. We denote by A_B the closed extension of A considered under the homogeneous boundary condition $Bu=0$. Under a certain condition posed on the pair (A, B) (cf. § 3), we construct the Green operator $(A_B+z)^{-1}$ in § 4. Our expression of the operator $(A_B+z)^{-1}$ enables us to know the asymptotic behaviour of $(A_B+z)^{-1}$ when z tends to infinity along ray of minimal growth introduced in Agmon [1]. Using this, we obtain the asymptotic expansion of Trace e^{-tA} when $t\to 0$ and of Trace $(A_B+\lambda)^{-1}$ when $\lambda\to\infty$. In the latter case, we of course assume that the order of A is larger than the dimension of M.

The behaviour of the pure imaginary power $A_B^{\kappa i}$ of A_B is, in general, very delicate even in L^2-theory. The simplest case is treated in § 6. If A is a single second order principally real operator and if B is the linear combination of the Neumann and the Dirichlet condition, then we can prove that $A_B^{\kappa i}$ is a bounded operator in L^p $(1<p<\infty)$ space and its norm can be estimated using the above results. This enables us to determine the domain $D(A_B^\theta)$ of fractional power A_B^θ $(0<\theta<1)$ of A_B in L^p space. If B includes derivatives which are tangential to ∂M, $A_B^{\kappa i}$ is, in general, unbounded except for $\kappa=0$ even in L^2 space.

All these results are obtained by using a special class of pseudo-differential operators treated in [6].

Results similar to those presented in § 2, § 4 and § 5 were announced by several authors (Seeley [18], Shimakura, Asano and Arima).