J. Math. Soc. Japan
Vol. 21, No. 3, 1969

On the convergence of nonlinear semi-groups II

By Isao MIYADERA[1)]

(Received Oct. 2, 1968)

§1. Introduction.

Let X be a Banach space and let $\{T(\xi); \xi \geqq 0\}$ be a family of (nonlinear) operators from X into itself satisfying the following conditions:

(i) $T(0)=I$ (the identity) and $T(\xi+\eta)=T(\xi)T(\eta)$ for $\xi, \eta \geqq 0$.

(ii) For each $x \in X$, $T(\xi)x$ is strongly continuous in $\xi \geqq 0$.

(iii) There is a constant $\omega \geqq 0$ such that

$$\|T(\xi)x-T(\xi)y\| \leqq e^{\omega\xi}\|x-y\|$$

for $x, y \in X$ and $\xi \geqq 0$.

We call such a family $\{T(\xi); \xi \geqq 0\}$ simply *nonlinear semi-group of local type*. In particular, if $\omega=0$, it is called a *nonlinear contraction semi-group*. We define the *infinitesimal generator* A_0 of $\{T(\xi); \xi \geqq 0\}$ by

$$A_0 x = \lim_{\delta \to 0+} \delta^{-1}(T(\delta)-I)x \tag{1.1}$$

and the *weak infinitesimal generator* A' by

$$A'x = w\text{-}\lim_{\delta \to 0+} \delta^{-1}(T(\delta)-I)x\,, \tag{1.2}$$

where the notation "w-lim" means the weak limit in X.

Throughout this paper it is assumed that the dual X^* of X is uniformly convex. Our purpose is to prove the following theorem.

THEOREM 1. *Let $\{T^{(k)}(\xi); \xi \geqq 0\}_{k=1,2,3,\cdots}$ be a sequence of nonlinear semi-groups of local type satisfying the stability condition*

$$\|T^{(k)}(\xi)x-T^{(k)}(\xi)y\| \leqq e^{\omega\xi}\|x-y\| \tag{1.3}$$

for $\xi \geqq 0$, k and $x, y \in X$, where ω is a non-negative constant independent of x, y, ξ and k. Let $A^{(k)}$ be the weak infinitesimal generator of $\{T^{(k)}(\xi); \xi \geqq 0\}$ and assume $R(I-h_k A^{(k)})=X$ for some $h_k \in (0, 1/\omega)$, and define $Ax=\lim_k A^{(k)}x$. Suppose that

(a) *$D(A)$ (the domain of A) is dense in X,*

(b) *$\overline{R(I-h_0 A)}=X$ for some $h_0 \in (0, 1/\omega)$,*

---

1) This work was partially supported by National Science Foundation Grant GP-8555.