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Differentiability of nonlinear semigroups

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In the previous paper [4] we discussed the Hille-Yosida theorem in case of nonlinear semigroups in Hilbert spaces: For a maximal dissipative operator A the evolution equation $\frac{d}{dt}u(t) \in A \cdot u(t)$, $u(0) \in D(A)$ has a unique solution in a certain weak sense, and hence such an operator generates uniquely a contraction semigroup, and conversely, if the generator A_0 of a contraction semigroup $\{T_t\}$ is densely defined, a maximal dissipative extension A of A_0 generates the initial semigroup $\{T_t\}$. Thus the following two problems have been left open:

1) whether weak solutions of $\frac{d}{dt}u \in A \cdot u$ for a maximal dissipative operator A are genuine solutions or not,

2) whether the generator of a nonlinear contraction semigroup in Hilbert space is densely defined or not.

In this paper we give positive answers to these problems. Further we study nonlinear holomorphic semigroups: We show a parallel theory with the linear case on such semigroups $\{T_t\}$'s that for fixed $x \in H$, $T_t x$ is holomorphic in t and for a fixed t, T_t is analytic as a mapping $H \to H$. Analytic mapping is a natural generalization of continuous linear operators.

In [3] Kato gave positive answer to the problem 1) in case of single-valued operator A, and extended main part of [4] to the case of Banach spaces with uniformly convex duals. He solved further nonlinear evolution equations in which the generator A depends on t. Some part of our results can be extended to the case of Banach spaces with uniformly convex duals or the case in which the generator A depends on t. For simplicity, however, we restrict ourselves to the case of nonlinear semigroups in Hilbert spaces.

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REMARK. After finishing this work the author was communicated by Professors Crandall, Pazy, Kato and Dorroh their new works [10], [11] and [13] which contain remarkable results. Especially, together with their results we attain to a complete form of the Hille-Yosida Theorem for nonlinear semi-