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A characterization of the simple group $S_p(6, 2)$

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§ 1. Introduction.

This is a continuation of our previous paper [11]. The purpose of this paper is to give a characterization of the finite simple group $S_p(6, 2)$, the symplectic group of 6 variables over the field of 2 elements, by the structure of the centralizer of an element of order 2 contained in the center of its Sylow 2-subgroup. Let V be a 6-dimensional vector space over the finite field $GF(2)$ and let f be a skew-symmetric non-degenerate bilinear form on V. The set of all non-singular linear transformations which leave f invariant form a group, the symplectic group over $GF(2)$. As is well-known, the structure of the symplectic group does not depend on the form f. So we may assume

$$f = x_1y_6 + x_2y_5 + x_3y_4 + x_4y_3 + x_5y_2 + x_6y_1 .$$

If J is the matrix of the form f, then the set of non-singular matrices A such that

$$^tAJA = J$$

may be identified with the symplectic group. Since this group has the trivial center, this is a simple group and of order $2^9 \cdot 3^4 \cdot 5 \cdot 7$ (cf. Artin [1]). Put

$$\hat{\alpha} = \begin{pmatrix} 1 & & & & & 1 \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}$$

and $\hat{H} = C(\hat{\alpha}) \cap S_p(6, 2)$. Then $\hat{\alpha}$ is a central involution of a Sylow 2-subgroup of $S_p(6, 2)$. Let A_n be the alternating group of degree n and $O_{2'}(G)$ be the maximal normal subgroup of odd order of the group G. Our main theorem of this paper is the following.

THEOREM. *Let G be a finite group such that G contains an element α of order 2 which is contained in the center of a Sylow 2-subgroup of G such that the centralizer $C_G(\alpha)$ is isomorphic to $\hat{H}$.*

Then (i) $G \cong A_{12}$ *or* A_{13} *or*

(ii) $G \cong S_p(6, 2)$ *or*