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A note on the large inductive dimension of totally normal spaces

Dedicated to Professor Atuo Komatu for his 60th birthday

By Keiô NAGAMI

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This paper gives a sum theorem, a characterization theorem, a product theorem and a coincidence theorem for the large (or equivalently strong) inductive dimension of totally normal spaces. The definitions and notations for the large and small inductive dimension can be seen in Nagata's dimension theory [**11**]. The concept of the total normality was introduced by C.H. Dowker in [**1**], offering a well designed class of spaces for considering the large inductive dimension, as follows: A space X is totally normal if i) X is normal and ii) for every open set G of X there exists a locally finite (in G) collection of cozero-sets $\{G_\alpha\}$ with $\cup G_\alpha = G$. We say that G_α is a cozero-set if there is a real-valued continuous function f_α defined on the whole space X with $G_\alpha = \{x : f_\alpha(x) \neq 0\}$. In [**1**] the following are proved:

(a) A hereditarily paracompact space is totally normal.

(b) A perfectly normal space is totally normal.

(c) There exists a totally normal space which is neither paracompact nor perfectly normal.

(d) Every subset of a totally normal space is totally normal.

(e) If X is a totally normal space, then the subset theorem and the sum theorem for the large inductive dimension are true as follows: i) If $Y \subset X$, then $\operatorname{Ind} Y \leqq \operatorname{Ind} X$. ii) If Y_i, $i=1, 2, \cdots$, are closed in X, then $\operatorname{Ind}(\cup Y_i) = \sup \operatorname{Ind} Y_i$.

All spaces considered in this paper are Hausdorff.

THEOREM 1. *Let X be a totally normal space having the weak topology*[1] *with respect to a closed covering $\{F_\alpha : \alpha \in A\}$. If $\operatorname{Ind} F_\alpha \leqq n$ for each $\alpha \in A$, then $\operatorname{Ind} X \leqq n$.*

PROOF (by double induction). The theorem is evidently true for the case:

1) According to K. Morita [7] a space X has the weak topology with respect to its closed covering $\{F_\alpha : \alpha \in A\}$ if the following condition is satisfied: A subset S of X is closed if and only if for an arbitrary subset B of A with $S \subset \bigcup \{F_\alpha : \alpha \in B\}$, $S \cap F_\alpha$ is closed for each $\alpha \in B$.