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Transformations of pseudo-Riemannian manifolds

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An m-dimensional pseudo-Riemannian manifold (M, g) is by definition a differentiable manifold M with a definite or indefinite Riemannian metric tensor g of signature (r, s). If the signature of g is $(m, 0)$, then we say that (M, g) is a Riemannian manifold. The purpose of this note is to generalize the results on transformations of Riemannian manifolds to those of pseudo-Riemannian manifolds.

In section 1 we give the basic relations of connections or various tensors satisfied by projective or conformal transformations. In section 2 we consider affine transformations and, for example, we get

COROLLARY 2.5. *If (M, g) is a compact irreducible pseudo-Riemannian manifold of signature (r, s) satisfying $r \neq s$, then any affine transformation of M is an isometry.*

In sections 3, 4, 5 and 6 we study projective and conformal transformations leaving some tensors invariant, in a similar way as in K. Yano and T. Nagano's paper [**10**]. However, some statements of theorems in [**10**] seem to be imcomplete, and so we give here complete statements and prove them in pseudo-Riemannian manifolds. For example we have

PROPOSITION 5.1. *Let (M, g) and $(N, 'g)$ be pseudo-Riemannian manifolds of dimension $m \geqq 4$. If there is a conformal transformation φ of M to N which leaves the covariant derivatives of the Weyl conformal curvature tensors invariant and if the set of points where φ is non-affine is dense in M, then M and N are conformally flat.*

As a consequence of this proposition we have

PROPOSITION 5.3. *Let M $(m \geqq 4)$ be an irreducible locally symmetric pseudo-Riemannian manifold of signature (r, s), $r \neq s$. Then we have either*

(i) *M is of constant curvature, or*

(ii) *M does not admit any non-homothetic conformal transformation.*

In the last section we give examples which support our statements of Proposition 3.1 and Proposition 5.1.