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On the integrability of Killing-Yano's equation

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Introduction.

In a Riemannian space M^n, a Killing vector v^h is a vector field satisfying the Killing's equation:

$$\nabla_i v_j + \nabla_j v_i = 0 ,$$

where ∇_i denotes the operator of the Riemannian covariant derivation. A Killing vector generates (locally) a one parameter group of isometries. On the other hand a one parameter group of affine transformations induces an affine Killing vector v^h characterized by the equation:

$$\nabla_j \nabla_i v^h + R_{lji}{}^h v^l = 0 .$$

K. Yano[1] have introduced a Killing tensor of order r as a skew symmetric tensor field $u_{i_1 \cdots i_r}$ satisfying

$$\nabla_{i_0} u_{i_1 \cdots i_r} + \nabla_{i_1} u_{i_0 i_2 \cdots i_r} = 0 .$$

In a previous paper[2], one of the authors discussed on Killing tensor of order 2. We shall generalize the results to the case of order $r \geqq 2$. In § 1 a system of linear differential equations to be satisfied by a Killing tensor is obtained. This equation enable us to define an affine Killing tensor as a generalization of an affine Killing vector. It will be shown that an affine Killing tensor is a Killing tensor in a compact M^n. We shall devote § 2 to prove that M^n is a space of constant curvature if it admits sufficiently many Killing tensors. § 3 deals with the converse problem. Thus we have a new characterization of a space of constant curvature. In § 4 we shall give examples of Killing tensor in the Euclidean space and the Euclidean sphere.

§ 1. Killing tensor. Affine Killing tensor.

Let M^n be an n dimensional Riemannian space whose metric tensor is given by g_{ab}[3] in terms of local coordinates $\{x^h\}$. We can regard the com-

1) K. Yano, [3].
2) S. Tachibana, [2].
3) $a, b, \cdots, i, j, \cdots = 1, \cdots, n$.