J. Math. Soc. Japan
Vol. 21, No. 2, 1969

On doubly transitive permutation groups of degree n and order $4(n-1)n$*

By Hiroshi KIMURA

(Received May 27, 1968)

§ 1. Introduction.

Doubly transitive permutation groups of degree n and order $2(n-1)n$ were determined by N. Ito ([4]).

The object of this paper is to prove the following result.

THEOREM. *Let Ω be the set of symbols $1, 2, \cdots, n$. Let $\mathfrak{G}$ be a doubly transitive group on Ω of order $4(n-1)n$ not containing a regular normal subgroup and let $\mathfrak{K}$ be the stabilizer of the set of symbols 1 and 2. Assume that $\mathfrak{K} \cap G^{-1}\mathfrak{K}G = 1$ or $\mathfrak{K}$ for every element G of $\mathfrak{G}$. Then we have the following results;*

(I) *If $\mathfrak{K}$ is a cyclic group, then $\mathfrak{G}$ is isomorphic to either $PGL(2, 5)$ or $PSL(2, 9)$.*

(II) *If K is an elementary abelian group, then $\mathfrak{G}$ is isomorphic to $PSL(2, 7)$.*

We use the standard notation. $C_{\mathfrak{X}}\mathfrak{T}$ denotes the centralizer of a subset $\mathfrak{T}$ in a group $\mathfrak{X}$ and $N_{\mathfrak{X}}\mathfrak{T}$ stands for the normalizer of $\mathfrak{T}$ in $\mathfrak{X}$. We denote the number of elements in $\mathfrak{T}$ by $|\mathfrak{T}|$.

§ 2. Proof of Theorem, (I).

1. Let $\mathfrak{H}$ be the stabilizer of the symbol 1. $\mathfrak{K}$ is of order 4 and it is generated by a permutation K whose cyclic structure has the form $(1)(2)\cdots$. Since $\mathfrak{G}$ is doubly transitive on Ω, it contains an involution I with the cyclic structure $(1\ 2)\cdots$. We may assume that I is conjugate to K^2. Then we have the following decomposition of $\mathfrak{G}$;

$$\mathfrak{G} = \mathfrak{H} + \mathfrak{H}I\mathfrak{H}\,.$$

Since I is contained in $N_{\mathfrak{G}}\mathfrak{K}$, it induces an automorphism of $\mathfrak{K}$ and (i) $\langle I\rangle\mathfrak{K}$ is an abelian 2-group of type $(2, 2^2)$ or (ii) $\langle I\rangle\mathfrak{K}$ is dihedral of order 8. If an element $H'IH$ of a coset $\mathfrak{H}IH$ of $\mathfrak{H}$ is an involution, then $IHH'I = (HH')^{-1}$ is contained in $\mathfrak{K}$. Hence, in case (i) the coset $\mathfrak{H}IH$ contains just two involutions,