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On the Cauchy problem for equations with multiple characteristic roots

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§ 1. Introduction.

Let us consider a kowalewskian

$$Lu=\left(\frac{\partial}{\partial t}\right)^m u+\sum_{\substack{|\nu|+j\leqq m\\ j\leqq m-1}} a_{\nu,j}(x,t)\left(\frac{\partial}{\partial x}\right)^{\nu}\left(\frac{\partial}{\partial t}\right)^j u=f \tag{1.1}$$

where

$$x=(x_1, x_2, \cdots, x_k)\in R^k,$$

$$\left(\frac{\partial}{\partial x}\right)^{\nu}=\left(\frac{\partial}{\partial x_1}\right)^{\nu_1}\left(\frac{\partial}{\partial x_2}\right)^{\nu_2}\cdots\left(\frac{\partial}{\partial x_k}\right)^{\nu_k},$$

$$|\nu|=\nu_1+\nu_2+\cdots+\nu_k$$

and

$$a_{\nu,j}(x,t)\in \mathcal{B}(R^{k+1})^{1)}.$$

We denote the principal part of L by L_0:

$$L_0=\left(\frac{\partial}{\partial t}\right)^m+\sum_{\substack{|\nu|+j=m\\ j\leqq m-1}} a_{\nu,j}(x,t)\left(\frac{\partial}{\partial x}\right)^{\nu}\left(\frac{\partial}{\partial t}\right)^j. \tag{1.2}$$

We look for a necessary condition in order that the Cauchy problem for (1.1) is well posed when (1.2) has multiple characteristic roots with constant multiplicity.

First we give

DEFINITION 1.1. The Cauchy problem for (1.1) is said to be well posed in L^2 sense in an interval $[0, T]$ if the following two conditions are satisfied.

(1) For any prescribed initial data Ψ

$$\Psi=\left\{\left(\frac{\partial}{\partial t}\right)^j u|_{t=0}=u_j\in \mathcal{D}_{L^2}^{m-j-1}, j=0, 1, 2, \cdots, m-1\right\} \tag{1.3}$$

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1) $\mathcal{B}(R^k)$ is the class of functions $f(x)=f(x_1, \cdots, x_k)$ such that their derivatives $\left(\frac{\partial}{\partial x}\right)^{\nu} f$ are bounded and continuous for $|\nu|=0, 1, 2, \cdots$.