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# On the alternating groups II

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## Introduction.

Let $\mathfrak{A}_m$ be the alternating group on $m$ letters $\{1, 2, \cdots, m\}$. Put $m=4n+r$, where $n$ is a positive integer and $0 \leqq r \leqq 3$. Let $\tilde{\alpha}_n$ be an involution of $\mathfrak{A}_m$ which has a cycle decomposition

$$(1, 2)(3, 4) \cdots (4n-3, 4n-2)(4n-1, 4n).$$

$\tilde{\alpha}_n$ is contained in the center of a 2-Sylow subgroup of $\mathfrak{A}_m$. For $r=1$, 2 and 3, we denote by $\tilde{H}(n, r)$ the centralizer in $\mathfrak{A}_m$ of $\tilde{\alpha}_n$. In the present paper, we shall prove the following two theorems.

THEOREM I. *Let $G(n, r)$ be a finite group with the following properties:*

(1) *$G(n, r)$ has no subgroup of index 2, and*

(2) *$G(n, r)$ contains an involution $\alpha_n$ in the center of a 2-Sylow subgroup of $G(n, r)$ whose centralizer $C_{G(n,r)}(\alpha_n)$ is isomorphic to $\tilde{H}(n, r)$.*

*Then if $r=2$ or 3, $G(n, r)$ is isomorphic to $\mathfrak{A}_{4n+r}$ except for the case $n=1$ and $r=2$ where $G(1, 2) \cong \mathfrak{A}_6$ or* PSL (2, 7).

For the case $r=1$, the author has not obtained the analogous result. But we can prove much weaker result. We note that $\tilde{H}(n, 1)$ has a unique elementary abelian subgroup $\tilde{S}$ of order $2^{2n}$ up to conjugacy (cf. Appendix, Proposition 5). Then we have

THEOREM II[(0)]. *Let $G(n, 1)$ be a finite group containing an involution whose centralizer $H(n, 1)$ is isomorphic to $\tilde{H}(n, 1)$. Let $S$ be an elementary abelian snbgroup of order $2^{2n}$ of $H(n, 1)$. Assume that there exists a one-to-one mapping $\theta$ from $\tilde{H}(n, 1) \cup N_{\mathfrak{A}_m}(\tilde{S})$ (the set theoretic union in $\mathfrak{A}_m$) onto $H(n, 1) \cup N_{G(n,1)}(S)$ such that $\theta$ induces an isomorphism between $\tilde{H}(n, 1)$ (resp. $N_{\mathfrak{A}_m}(\tilde{S})$) and $H(n, 1)$ (resp. $N_{G(n,1)}(S)$).*

*Then $G(n, 1)$ is isomorphic to $\mathfrak{A}_{4n}$ or $\mathfrak{A}_{4n+1}$.*

The proof of Theorem I depends on Theorem A of the author's previous paper [**9**] which was proved only in the case $r=2$ or 3. But we have not obtained such result for the case $r=1$. This is the reason why the stronger condition is necessary for the case $r=1$. However, we note: Theorem II shows that, if we can prove a result in the case $r=1$ similar to Theorem A of [**9**], we shall be able to at once obtain a characterization of $\mathfrak{A}_{4n}$ and $\mathfrak{A}_{4n+1}$ under