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# A remark on the cohomology group and the dimension of product spaces

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**1.** In § 4 of the paper [4] several theorems concerning the dimension of product spaces were given. Proofs of all theorems except Theorem 5 depend heavily on Künneth formula which was proved by R. C. O'Neil [7]. However this formula is false. It is known by a counter example given by G. Bredon (see **2**). The purpose of this paper is devoted to correct some of theorems in § 4 of [4] and to prove the related results. However it is not known whether Theorems 6–9 hold or not though they are proved partly in this paper.

Throughout this paper all spaces are Hausdorff and have finite covering dimension and we mean by $H^*$ the unrestricted Čech cohomology group.

**2.** R. C. O'Neil [7] gave the following theorems.

**A.** Let $G$ be an abelian group. If $X\times Y$ is paracompact, then

$$H^n(X\times Y:G)\cong\sum_{q=0}^{n}H^q(X:H^{n-q}(Y:G)).$$

**B.** Let $L$ be a principal ideal domain. If $X$ is compact and $Y$ is paracompact, then there is an exact sequence

$$0\rightarrow\sum_{q=0}^{n}H^q(X:L)\underset{L}{\otimes}H^{n-q}(Y:L)\rightarrow H^n(X\times Y:L)$$

$$\rightarrow\sum_{q=0}^{n}H^{q+1}(X:L)*_L H^{n-q}(Y:L)\rightarrow 0.$$

The following example was given by G. Bredon. Let $X$ be a solenoid, so $X$ is a 1-dimensional compact metric space and $H^1(X)\cong R$ (=the group of all rational numbers). For $n=2, 3, \cdots$, let $Y_n$ be a 2-dimensional finite simplicial polytope such that $H^2(Y_n)\cong Z_n$ (=the cyclic group of order $n$). Let $Y$ be a disjoint union of $Y_n$, $n=2, 3, \cdots$. Then $Y$ is a 2-dimensional locally finite polytope and $H^2(Y)\cong\prod_{n=2}^{\infty}Z_n$. Since $X$ is compact, by Peterson [8: Appendix], we have $H^3(X\times Y)\cong\prod_{n=2}^{\infty}H^3(X\times Y_n)\cong\prod_{n=2}^{\infty}H^1(X)\otimes H^2(Y_n)=0$ and $H^1(X:H^2(Y))\cong H^1(X)\otimes H^2(Y)\neq 0$. Thus, both theorems A and B are false.