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On 12-manifolds of a special kind

Dedicated to Professor Atuo Komatu on his 60th birthday

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§1. Preliminary.

Let K be a simply connected CW-complex whose cohomology groups are as follows

$$H^0(K)=H^4(K)=H^8(K)=H^{12}(K)\cong Z \quad \text{and} \quad H^i(K)=0 \quad \text{for other } i.$$

In this paper we shall consider conditions under which K has a homotopy type of a compact C^∞-manifold without boundary. By using Novikov-Browder theory[1] we can partially solve the above problem. By an orientation of K we mean a pair of generators of $H^8(K)$ and $H^{12}(K)$[2]. Since K is homotopy equivalent to a CW-complex $S^4\cup e^8\cup e^{12}$ we can associate with K elements $\alpha\in\pi_7(S^4)$ and $\beta\in\pi_{11}(S^4\cup e^8)$ which are ∂-images of the generators of $\pi_8(K, S^4)$ and $\pi_{12}(K, S^4\cup e^8)$ carried by the orientation of K respectively. Here ∂ denotes the boundary homomorphism: $\pi_8(K, S^4)\to\pi_7(S^4)$ and $\pi_{12}(K, S^4\cup e^8)\to\pi_{11}(S^4\cup e^8)$ respectively. Let $h: S^7\to S^4$ be the Hopf map and let τ be the element of $\pi_7(S^4)$ such that $2[h]+\tau=[\iota_4, \iota_4]$[3]. It is known that $\pi_7(S^4)$ is isomorphic to the direct sum of Z and Z_{12} which are generated by $[h]$ and τ respectively. Hence we can replace α by two integers a, b $(0\leqq b\leqq 11)$ such that $\alpha=a[h]+b\tau$. In this paper the case $b=0$ shall be treated in which case we can replace β by numerical invariants. Let K_a be the CW-complex which is obtained by attaching e^8 to S^4 by a representative of $a[h]$, and let $\varphi_a: K_a\to K_1$ be a map which is the identity on S^4 and of degree a on e^8. Obviously, $K_1=P_2(Q)$, the quaternion projective plane. Denote by ξ_1 $(S^{11}\to P_2(Q)=K_1)$ the canonical S^3-bundle. Then let ξ_a be the bundle induced by φ_a, and by the same symbol ξ_a we denote also the total space of this bundle. We consider the group $\pi_{11}(K_a)$ and the diagram

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1) Concerning Browder's theorem and another application, see [**3**] and [**5**].
2) We suppose that an orientation of e^4 is fixed.
3) $[f]$ denotes the homotopy class of f, and $[\ ,\ ]$ Whitehead product.