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Diffeomorphism groups and classification of manifolds

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§ 0. Introduction.

The purpose of this paper is to investigate the groups of the pseudo-diffeotopy classes of diffeomorphisms of manifolds, which are total spaces of disk bundles over spheres or sphere bundles over spheres. The results are applied to the diffeomorphism classification of simply-connected manifolds, which are homological tori.

Let Diff M denote the group of orientation preserving diffeomorphisms of an oriented manifold M and let $\tilde{\pi}_0$ (Diff M) denote the group of pseudo-diffeotopy classes of Diff M. Let $\mathcal{E}_f$ and $\mathcal{F}_f$ be the D^{q+1} bundle over S^p and S^q bundle over S^p with characteristic map $f: S^{p-1} \to \mathrm{SO}_{q+1}$. In § 1, we study $\tilde{\pi}_0$ (Diff $\mathcal{E}_f$). In case where $\mathcal{E}_f = S^p \times D^{q+1}$, we prove the following theorem.

THEOREM 1.5. *Let $p < 2q-1$. The order of $\tilde{\pi}_0$* (Diff $S^p \times D^{q+1}$) *is equal to the order of the direct sum group π_p* $(\mathrm{SO}_{q+1}) \oplus \boldsymbol{Z}_2$.

The concordance classes of (framed) embeddings of S^q in $\mathcal{F}_f$ are discussed in § 2. The set of framed embedding classes are related to the pairing

$$\mathrm{F}: \pi_{p-1}(\mathrm{SO}_q) \times \pi_q(S^p) \to \pi_{q-1}(\mathrm{SO}_p)$$

introduced by Wall [14]. In § 3, we define a map C from $\tilde{\pi}_0$ (Diff $S^p \times S^q$) to Θ^{p+q+1} and study its properties. Making use of the results of § 1~3, the study of $\tilde{\pi}_0$ (Diff $\mathcal{F}_f$) is carried out in § 4. In case $\mathcal{F}_f = S^p \times S^q$, we obtain the following theorem.

THEOREM 4.17. *For $p < q < 2p-4$, the order of $\tilde{\pi}_0$* (Diff $S^p \times S^q$) *is equal to the order of the direct sum group*

$$\boldsymbol{Z}_2 \oplus \pi_p(\mathrm{SO}_{q+1}) \oplus \pi_q(\mathrm{SO}_{p+1}) \oplus \Theta^{p+q+1}.$$

In § 5, as an application of our results in § 4, we deal with the classification of manifolds which satisfy the conditions,

$$\left\{\begin{array}{l} M: \text{closed and simply connected} \\ \mathrm{H}_i(M) = \begin{cases} \boldsymbol{Z} & \text{for } 0,\ p,\ q+1,\ p+q+1 \\ 0 & \text{otherwise} \end{cases} \\ \pi_p(\mathrm{SO}_{q+1}) = 0 \\ p < q < 2p-4. \end{array}\right\} \qquad (*)$$