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On finite groups with a 2-Sylow subgroup isomorphic to that of the symmetric group of degree $4n$

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§ 0. Introduction.

Let G be a finite group with a 2-Sylow subgroup isomorphic to that of the symmetric group of degree $4n$. The purpose of the present paper is to make some remarks on the fusion of involutions of G, which are useful for the investigations of certain finite simple groups, especially the alternating group of degree $4n+2$ or $4n+3$ and the orthogonal commutator groups $\Omega_{2n+2}(\varepsilon, q)$ ($q^{n+1} \equiv -\varepsilon \bmod 4$ and $q \equiv \pm 3 \bmod 8$)[1].

The main results are Theorem A and Theorem B in § 7. We note that the Thompson subgroup of a 2-Sylow subgroup of G plays the important role in the discussions in § 2~§ 6. These can be regarded as a generalization of a part of [**6**]. Moreover, as an application of Theorem A, the author has obtained a characterization of the alternating groups of degrees $4n+2$ and $4n+3$ in terms of the centralizer of an involution $(1, 2)(3, 4)\cdots(4n-1, 4n)$. This will be published in a subsequent paper. Also H. Yamaki [**9**] has treated such characterizations of $\mathfrak{A}_m$ ($m=12$, 13, 14 and 15), though, for $m=12$ and 13, Theorem A can not be applied and an additional condition is necessary on account of the existence of the finite simple group $Sp_6(2)$.

Notations and Terminology.

$J(X)$	The Thompson subgroup of a group X (cf. [8])[2]
$Z(X)$	the center of a group X
X'	the commutator subgroup of X
$X \wr Y$	a wreath product of a group X by a permutation group Y
$x \sim y$ in X	x is conjugate to y in a group X
y^x	$x^{-1}yx$
$x: y \rightarrow z$	$y^x = z$
$[x, y]$	$x^{-1}y^{-1}xy$

1) For the notations of orthogonal groups, see [1] and [10]. Note that if $q^{n+1} \equiv -\varepsilon \bmod 4$, $\Omega_{2n+2}(\varepsilon, q)$ has the trivial center.

2) Recently, the slightly different definition of $J(X)$ from that of [8] is used, but for groups treated in the present paper, both definitions are the same.