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Homogeneous complex hypersurfaces

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In this paper we classify those complex hypersurfaces of a complex space form which are homogeneous spaces with respect to the induced Kähler structure. This is achieved in Theorem 2 and we may observe that the local classification is the same as that obtained for complex hypersurfaces with parallel Ricci tensor (see Theorem 4 [**4**]). In fact, Theorem 1 is a local result from which both classifications follow immediately. In proving Theorem 1 we need only draw on some of the basic properties of complex hypersurfaces, as developed in [**5**], and the results on the holonomy of complex hypersurfaces of § 2 [**4**].

While Theorem 1 contains the classification theorems of Chern [**1**], Nomizu and Smyth [**4**], and a result of Takahashi [**6**], it should be noted that Kobayashi [**3**] recently obtained a stronger result in the case where the ambient space is complex projective space, to wit: any complete complex hypersurface of constant scalar curvature in $P^{n+1}(C)$ is a projective hyperplane or a quadric.

The questions examined here arose from discussions with Professor K. Nomizu, for whose suggestions I am very grateful.

Let M be a complex n-dimensional manifold and let ϕ be a complex immersion of M in a Kähler manifold $\tilde{M}$ of complex dimension $n+1$ and constant holomorphic sectional curvature $\tilde{c}$. The Riemannian metric g induced on M by ϕ is a Kähler metric and all metric properties of M refer to this metric. M will be called *homogeneous* (Riemannian) if the group of isometries of M acts transitively on M; we remark that it will not be necessary to assume that M is homogeneous Kählerian to obtain Theorem 2. To each field ξ of unit vectors normal to M (with respect to the immersion ϕ) on a neighborhood $U(x_0)$ of a point $x_0 \in M$ there is associated a symmetric tensor field A of type (1, 1) on $U(x_0)$; A^2 is independent of the choice of ξ [**5**]. We shall use the same notation as in [**5**].

LEMMA 1. *The characteristic roots of A^2 are constant in value and multiplicity on M if either*

a) *M is homogeneous*

or b) *the Ricci tensor of M is parallel.*

PROOF. a) The Ricci tensor S of M is given by

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