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The prolongation of the holonomy group

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In a series of recent papers [2], Kobayashi and Yano [2; I] have defined a mapping from the tensor algebra of a manifold M into the tensor algebra of its tangent bundle $T(M)$. This mapping they called the "complete lift". They have also defined the complete lift of a connection on M to a connection on $T(M)$. In [2; III], they have shown that the holonomy group of the connection on $T(M)$ is the tangent group of the holonomy group of the connection on M. They mention that it should be possible to prove this in the spirit of [2; I]. The purpose of this paper is to compare the infinitesimal holonomy groups of M and $T(M)$ (see Nijenhuis [3] for definition and properties).

We will suppose that the manifold M is connected and analytic and also that the connection is analytic. In this case, Nijenhuis [3] has shown that the dimension of the infinitesimal holonomy group is constant on M and thus the infinitesimal holonomy group is equal to the restricted holonomy group of M. The main theorem of this paper then tells us that if the dimension of the Lie algebra of the holonomy group of M is r, then the dimension of the Lie algebra of the holonomy group of $T(M)$ is $2r$ and furthermore, it has an abelian ideal of dimension r. The result of [2; III] for M can easily be seen by the constructions contained here.

§ 1. Preliminaries.

Let M be a connected, analytic manifold of dimension n and $\mathfrak{X}(M)$ the module of vector fields on M. The connection will be denoted by ∇ and the covariant derivative operator by $\nabla_X (X \in \mathfrak{X}(M))$. Let R denote the curvature tensor of ∇. ∇ is assumed to be analytic. If (x^i) is a local coordinate system on M, let the corresponding coordinate system on $T(M)$ (the tangent bundle of M) be denoted by (x^i, y^i). Here we have $i=1, \cdots, n$.

Let $\pi : T(M) \to M$ be the natural projection map. Then, following Kobayashi and Yano [2], we define two mappings from the tensor algebra of M into the tensor algebra of $T(M)$. The first is called the "vertical lift", and is characterized by