J. Math. Soc. Japan
Vol. 20, No. 4, 1968

Mixed problems for hyperbolic equations of second order

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(Received Feb. 1, 1968)

§ 1. Introduction.

This paper is concerned with the mixed problems for hyperbolic equations of second order. Let S be a sufficiently smooth compact hypersurface in R^n, and let Ω be the interior or exterior domain of S.

Consider the hyperbolic equation of second order

$$L[u]=\frac{\partial^2}{\partial t^2}u+a_1(x,t:D)\frac{\partial}{\partial t}u+a_2(x,t:D)u=f \tag{1.1}$$

$$a_1(x,t:D)=\sum_{i=1}^{n}2h_i(x,t)\frac{\partial}{\partial x_i}+h(x,t)$$

$$a_2(x,t:D)=-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}(x,t)\frac{\partial}{\partial x_j}\right)+\sum_{j=1}^{n}b_j(x,t)\frac{\partial}{\partial x_j}+c(x,t)$$

where the coefficients belong to $\mathscr{B}^2(\Omega\times(-\delta_0,\infty))$[1]. We assume that $a_2(x,t:D)$ is an elliptic operator satisfying

$$\sum_{i,j=1}^{n}a_{ij}(x,t)\xi_i\xi_j>d\sum_{i=1}^{n}\xi_i^2 \qquad (d>0) \tag{1.2}$$

$$a_{ij}(x,t)=a_{ji}(x,t)$$

for all $(x,t)\in\Omega\times(-\delta_0,\infty)$ and $\xi=(\xi_1,\xi_2,\cdots,\xi_n)\in R^n$, and that $h_i(x,t)$ $(i=1,2,\cdots,n)$ are real-valued. For this equation we consider the following boundary conditions

$$B_1u(x,t)=u(x,t)=0 \quad \text{on} \quad S, \tag{1.3}$$

$$B_2u(x,t)=\frac{\partial}{\partial n_t}u(x,t)-\langle h,\nu\rangle\frac{\partial}{\partial t}u(x,t)+\sigma(s,t)u(x,t)=0 \quad \text{on} \quad S \tag{1.4}$$

where

$$\frac{\partial}{\partial n_t}=\sum_{i,j=1}^{n}a_{ij}(s,t)\nu_i\frac{\partial}{\partial x_j}, \qquad \langle h,\nu\rangle=\sum_{i=1}^{n}h_i(s,t)\nu_i,$$

$\nu=(\nu_1,\cdots,\nu_n)$ is the outer unit normal of S at $s\in S$, and $\sigma(s,t)$ is a real-valued

1) $\mathscr{B}^k(\omega)$, ω being an open set, is the set of all functions defined in ω such that their partial derivatives of order $\leqq k$ all exist and are continuous and bounded.