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On certain square-integrable irreducible unitary representations of some $\mathfrak{p}$-adic linear groups

By Takuro SHINTANI

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Introduction.

Let P be a $\mathfrak{p}$-adic number field. Denote by $\mathfrak{O}$, $\mathfrak{P}$ and $\mathfrak{O}^*$ the ring of integers, the maximal ideal of $\mathfrak{O}$ and the unit group respectively. We consider the subgroup G of $GL(n^{1)}, P)$ formed by matrices with determinant in $\mathfrak{O}^*$. $K=GL(n, \mathfrak{O})$ is a maximal compact subgroup of G. In this paper, we construct continuous irreducible unitary representations of K parametrized by certain characters (which must satisfy rather restrictive conditions) of compact Cartan subgroups of G. We then show that unitary representations of G induced by these irreducible representations of K are irreducible and square integrable.

This paper is divided into four sections and last two sections are divided into several subsections. In § 1, we prove results in the theory of induced representations of finite groups, which are basic in our argument. In § 2, using results of § 1, we show that continuous irreducible unitary representations of K, which are not reduced to representations of $GL(n, \mathfrak{O}/\mathfrak{P})$, are induced by certain irreducible representations of some subgroups of K (Theorem 1). We further show that there exists a rather large family of irreducible unitary representations of K which are monomial (Theorem 2). In § 3, we study unitary representations of G induced by irreducible unitary representations of K. After the study of general properties of such representations, we show that there exists a rather large family of irreducible unitary representations of K which induce square integrable irreducible unitary representations of G (Theorem 3). We also show that an analogue of Frobenius' formula for induced characters is valid. In § 4, we first study correspondence between compact Cartan subgroups of G and extensions of P of degree n. Then, in 4-4 and 4-5, we construct irreducible unitary representations of K parametrized by certain characters, which satisfy certain conditions ('strong regularity' in our terminology), of compact Cartan subgroups of G. We show that they induce square integrable irreducible unitary representations of G (Theorem 4). Theo-

1) n is a natural number $\geqq 2$.