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Differential geometry of complex hypersurfaces II*

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In this paper we continue the study of complex hypersurfaces of complex space forms (i. e. Kählerian manifolds of constant holomorphic sectional curvature) begun in [8]. The main results are: the determination of the holonomy groups of such hypersurfaces, a generalization of the main theorem of [8] on Einstein hypersurfaces, the non-existence of a certain type of hypersurface in the complex projective space, and some results concerning the curvature of complex curves.

Let $\tilde{M}$ be a complex space form (which in general will not be complete) of complex dimension $n+1$ and let M be an immersed complex hypersurface in $\tilde{M}$. In § 1 we show that the rank of the second fundamental form of M is intrinsic and that M is rigid in $\tilde{M}$, if the latter is simply connected and complete. The local version of rigidity is contained as a special case in the work of Calabi [1], but our method is more direct and more in the line of classical differential geometry.

The holonomy group of M (with respect to the induced Kähler metric) is studied in § 2. If the holomorphic sectional curvature $\tilde{c}$ of $\tilde{M}$ is negative, the holonomy group is always $U(n)$. In the case where $\tilde{c}>0$ (e. g. $\tilde{M}=P^{n+1}(C)$), the holonomy group of M is either $U(n)$ or $SO(n)\times S^1$ (S^1 denotes the circle group), the latter case arising only when M is locally holomorphically isometric to the complex quadric Q^n in $P^{n+1}(C)$. When $\tilde{c}=0$ (i. e. when $\tilde{M}$ is flat), the holonomy group of M depends on the rank of the second fundamental form and we obtain a result of Kerbrat [3] more directly.

In § 3 we first obtain the following generalized local version of the classification theorem of [8]. If the Ricci tensor S of M is parallel (i. e. $\nabla S=0$), then M is totally geodesic in $\tilde{M}$ or else $\tilde{c}>0$ and M is locally a complex quadric. To prove this we modify Theorem 2 [8] to show that M is locally symmetric when its Ricci tensor is parallel, and obtain the local classification without using the list of irreducible Hermitian symmetric spaces. This local version was proved by Chern [2] with the original assumption that M is Einstein, and Takahashi [9] has shown that M is Einstein if its Ricci tensor

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