J. Math. Soc. Japan
Vol. 20, Nos. 1-2, 1968

On the pre-closedness of the potential operator

Dedicated to Professor Iyanaga on his 60th birthday

By Kôsaku YOSIDA, Takeshi WATANABE
and Hiroshi TANAKA

(Received Oct. 30, 1967)

§1. **Introduction.** Let X be a separable, locally compact, non-compact Hausdorff space, and B be the completion with respect to the maximum norm of the space $C_0(X)$ of real-valued continuous functions with compact supports defined in X. G. A. Hunt [1] introduced the notion of the potential operator V as a positive linear operator on $D(V) \subseteqq B$ with $D(V) \supseteqq C_0(X)$ into B satisfying the "principle of positive maximum"[1]:

(1) For any $f \in C_0(X)$, we have $\sup_{f(x)>0} (Vf)(x) = \sup_{x \in X} (Vf)(x)$ if the latter supremum is positive.

The fundamental result of Hunt reads as follows:

THEOREM. *Let V satisfy* (1) *and the condition that*

(2) $V \cdot C_0(X)$ *is dense in* B.

Then, there exists a uniquely determined semi-group $\{T_t; t \geqq 0\}$ *of class* (C_0) *of positive contraction linear operators* T_t *on* B *into* B *such that*

(3) $AVf = -f$, $f \in C_0(X)$, *for the infinitesimal generator* A *of* T_t.

An operator-theoretical proof of this theorem was given in K. Yosida [2], showing that the resolvent $J_\lambda = (\lambda I - A)^{-1}$, $\lambda > 0$, of A is the continuous extension to the whole space B of the operator $\hat{J}_\lambda$ defined by

$$\lambda Vf + f \to Vf, \qquad f \in C_0(X), \tag{4}$$

with an additional remark that

(5) V^{-1} exists and $V^{-1} = -A$ if and only if V is closed.

The purpose of the present note is to show that *the restriction $V|C_0(X)$ of V to $C_0(X)$ is pre-closed so that its smallest closed extention, which shall be*

---

1) This principle, sometimes called as the "weak principle of positive maximum", is proved on page 220 of [2] in the course of the proof of:

(1)′ For any $f \in C_0(X)$, the condition $(Vf)(x_0) = \sup_{x \in X} (Vf)(x)$ implies $f(x_0) \geqq 0$.

It is also proved on the same page that (1)′ is a consequence of (1) and (2).