J. Math. Soc. Japan
Vol. 20, Nos. 1-2, 1968

On the classification of sufficiently connected manifolds

Dedicated to Professor S. Iyanaga on his 60th birthday

By Itiro TAMURA

(Received Aug. 29, 1967)

Smale theory enables us to give a handlebody presentation for a simply-connected closed oriented differentiable m-manifold ($m \geqq 6$) and to establish a diffeomorphism between two such manifolds by proving that they are h-cobordant. In consequence we can achieve the determination of differentiable manifolds with a certain homotopy type, by firstly fixing up explicit handlebody presentations and secondly considering h-cobordism classes among them.

In this note we perform this for simply-connected closed oriented differentiable m-manifolds such that homology groups are trivial except in dimensions k, $m-k$ and for which certain cohomology operations vanish, where $m=2n$, $k=n-1$, $n \geqq 6$, or $m=2n+1$, $k=n-1$, $n \geqq 7$.

Chief results will be stated as Theorems 1, 2 and 3. However, in order to have a proper understanding of the form of our theorems, we pick up here some results in them:

Let M^{2n} be a simply connected closed oriented differentiable $2n$-manifold ($n \geqq 6$, $n \equiv 4, 6, 7 \bmod 8$) such that homology groups are trivial except in dimensions $n-1$, $n+1$ and that $Sq^2(H^{n-1}(M^{2n}; Z_2))=0$. Then M^{2n} is diffeomorphic to a connected sum of an S^{n-1}-bundle over S^{n+1}, copies of $S^{n-1} \times S^{n+1}$ and a homotopy sphere. In case $n \equiv 7 \bmod 8$, this presentation is unique up to diffeomorphism.

Let M^{2n+1} be a simply connected closed oriented differentiable $(2n+1)$-manifold ($n \geqq 7$, $n \equiv 6, 7 \bmod 8$) such that homology groups are trivial except in dimensions $n-1$, $n+2$ and that $\Phi(H^{n-1}(M^{2n+1}; Z_2))=0$. Then M^{2n+1} is diffeomorphic to a connected sum of an S^{n-1}-bundle over S^{n+2}, copies of $S^{n-1} \times S^{n+2}$ and a homotopy sphere. In case $n \equiv 6 \bmod 8$, this presentation is unique up to diffeomorphism.

1. Presentations.

Let M^m be a simply-connected closed oriented differentiable m-manifold ($m \geqq 7$) such that

This research was partially supported by the D. S. I. R.