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Algebraic varieties without deformation and the Chow variety

Dedicated to Professor Shôkichi Iyanaga on his 60th birthday

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The purpose of this note is to indicate two simple facts, of which the first one is almost obvious, once it is formulated:

THEOREM 1. *If a complex projective non-singular variety V has no deformation, then V is biregularly equivalent to a projective variety defined over an algebraic number field.*

Here we say that V has *no deformation* if $H^1(V, \Theta)=0$, where Θ denotes the sheaf of germs of holomorphic sections of the tangent bundle of V. Calabi and Vesentini [**1**] have proved that $H^1(V, \Theta)=0$ if V is the quotient S/Γ of an irreducible bounded symmetric domain S of dimension >1 by a discontinuous group Γ operating freely on S such that S/Γ is compact. (See also [**4**], [**5**], [**11**].) Therefore Th. 1 shows that such a quotient has a model defined over an algebraic number field.

To state the second fact, let $C_p(N, m, d)$ denote the set of all the Chow points of positive cycles of dimension m and degree d in the projective space of dimension N, defined with respect to a universal domain of characteristic $p \geqq 0$. It is well known that $C_p(N, m, d)$ is a Zariski closed set in a certain projective space, which is defined by equations with coefficients in the prime field. Then one can ask the following question:

(Q) *Is every absolutely irreducible component of $C_p(N, m, d)$ defined over the prime field?*[1]

The answer is negative if the characteristic is 0:

THEOREM 2. *There exist positive integers N, m, d such that $C_0(N, m, d)$ has a component which is not defined over the rational number field.*

Such a component will be obtained so as to contain the Chow point of a certain variety without deformation. This is why we present these two theorems together. We shall also show in the last section that the answer to the question (Q) is still negative even if the characteristic is positive.

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1) I thank S. Lichtenbaum for reminding me of this question.