J. Math. Soc. Japan
Vol. 20, Nos. 1-2, 1968

On a certain invariant of the groups of type E_6 and E_7

Dedicated to Professor S. Iyanaga on his 60th birthday

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(Received Aug. 10, 1967)

In my recent paper [**9**], I have introduced an invariant $\gamma(G)$ for a connected semi-simple algebraic group G, which generalizes the classical invariants of Hasse and of Minkowski-Hasse, and have shown that, for a classical simple group G, $\gamma(G)$ can actually be determined explicitly in terms of these classical invariants[1]. For exceptional groups, however, I gave only a very brief indication for the case where the ground field is a local field or an algebraic number field ([**9**], 250-251). The purpose of this note[2] is to give a more comprehensive account for a more general case, establishing a principle which enables us to reduce the determination of $\gamma(G)$ for an exceptional group G to that for a suitably chosen *classical* subgroup G' of G defined over the same ground field. The existence of such a subgroup G' will be ascertained for the groups of type E_6 and E_7 constructed recently by Tits [**12**].

1. Throughout this paper, k is a field of characteristic zero, (though it seems likely that most of our results remain true over any perfect field of characteristic different from 2 and 3). $\bar{k}$ is a fixed algebraic closure of k and $\mathfrak{G}=\mathrm{Gal}(\bar{k}/k)$ is the Galois group of $\bar{k}/k$ operating on $\bar{k}$ from the right. For an algebraic group G defined over k, we write the Galois cohomology set or group $H^i(\mathfrak{G}, G_{\bar{k}})$ $(i=1, 2)$ as $H^i(k, G)$. $\mathbf{E}_n=\{\zeta_n\}$ is the group of all n-th roots of unity contained in $\bar{k}$. In principle, we follow the notation in [**9**].

Let G_1 be an algebraic group defined over k. By an *inner k-form* of G_1,

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*) Partially supported by NSF grant GP-6654.

1) Taking this opportunity, I would like to correct some of the misprints in the relevant part of [**9**]. On page 246, line 10, for "$\mathfrak{R}^{\Sigma m_i}$" read "$\mathfrak{R}^{\Sigma i m_i}$"; similar corrections are also necessary for the formulas (28), (28′) in page 250. On page 249, line 9, for "$k(\sqrt{(-1)^{1/2\,nr}}\det(S))$" read "$k(\sqrt{(-1)^{1/2\,nr}\det(S)})$"

2) By a communication from Professor Tits, the author learnt after completion of the paper that similar topics had also been treated by him in a series of lectures delivered at Yale University in the winter of 1967.

Added in proof: By a communication with Tits, it appeared that in 8 the relation $\mathfrak{C}_2\sim\mathfrak{D}'$ and so (11) is always true without any assumption.