J. Math. Soc. Japan
Vol. 20, Nos. 1-2, 1968

On the generalized decomposition numbers of the symmetric group

Dedicated to Professor Iyanaga on his 60th birthday

By Masaru OSIMA

(Received Aug. 30, 1967)

Introduction

Let G be a group of finite order and let p be a fixed prime number. We consider the representations of G in the field Ω of the g-th roots of unity. Then every absolutely irreducible representation of G can be written with coefficients in Ω. Let $\mathfrak{p}$ be a prime ideal divisor of p in Ω and let $\mathfrak{o}_{\mathfrak{p}}$ be the ring of all $\mathfrak{p}$-integers of Ω, and Ω^* the residue class field of $\mathfrak{o}_{\mathfrak{p}}$ (mod $\mathfrak{p}$). We denote by α^* the residue class of $\alpha \in \mathfrak{o}_{\mathfrak{p}}$.

Let $\zeta_0 = 1, \zeta_1, \cdots, \zeta_{m-1}$ be the (absolutely) irreducible characters of G and let $\varphi_0 = 1, \varphi_1, \cdots, \varphi_{n-1}$ be the modular irreducible characters of G for p. Then we have for a p-regular element y in G

$$\zeta_i(y) = \sum_{\kappa} d_{i\kappa}\varphi_\kappa(y) \tag{1}$$

where the $d_{i\kappa}$ are non-negative rational integers and are called the decomposition numbers of G. The irreducible characters ζ_i and the modular irreducible characters φ_κ are distributed into a certain number of blocks $B_0, B_1, \cdots, B_{s-1}$ for p, each ζ_i and each φ_κ belonging to exactly one block B_σ. In (1) we have $d_{i\kappa} = 0$ for $\zeta_i \in B_\sigma$ if φ_κ is not contained in B_σ.

In the following we denote by x the p-element of G. Let $\varphi_0^x = 1, \varphi_1^x, \cdots, \varphi_{r-1}^x$ be the modular irreducible characters of the normalizer $N(x)$ of x in G. We have for a p-regular element y in $N(x)$

$$\zeta_i(xy) = \sum_{\kappa} d_{i\kappa}^x \varphi_\kappa^x(y) \tag{2}$$

where the $d_{i\kappa}^x$ are the algebraic integers and are called the generalized decomposition numbers of G. We have $d_{i\kappa} = d_{i\kappa}^1$ for $x = 1$. Let us denote by $B^{(\sigma)}$ the collection of all blocks $\tilde{B}_\tau$ of $N(x)$ which determine a given block B_σ of G. In (2) we have $d_{i\kappa}^x = 0$ for $\zeta_i \in B_\sigma$ if φ_κ^x is not contained in $B^{(\sigma)}$ ([1], [3]).

Recently A. Kerber [5] proved the following

THEOREM 1. *The generalized decomposition numbers of the symmetric group*