J. Math. Soc. Japan
Vol. 20, Nos. 1-2, 1968

A mean value theorem in adele geometry

Dedicated to Professor S. Iyanaga for his 60th birthday

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(Received July 10, 1967)

In this paper, we point out that one can test the validity of the mean value theorem ([**10**], [**11**]) for the adele transformation spaces attached to certain algebraic transformation spaces defined over the rationals by looking at the first two homotopy groups of the underlying complex manifolds.

Notation and conventions: As usual $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{C}$ are the integers, the rational numbers, the real numbers, the complex numbers, respectively. Further, we shall frequently use the following notation

$\boldsymbol{Q}_v$: the completion of $\boldsymbol{Q}$ at a valuation v of $\boldsymbol{Q}$.
$\boldsymbol{Z}_p$: the p-adic integers in $\boldsymbol{Q}_p$.
$\boldsymbol{F}_p$: the finite field with p elements.
S: any finite set of valuations of $\boldsymbol{Q}$ including $v=\infty$.
Ω: a universal domain.
G_m: the multiplicative group of Ω.
K^*: the multiplicative group of a field K.
$\bar{K}$: the algebraic closure of a field K.
$[E]$: the cardinality of a set E.

If O denotes a set of geometric objects, we shall denote by O_K the subset of O which is rational over a field K. For a topological space X, we shall denote by $L(X)$ the set of all $\boldsymbol{R}$-valued continuous functions on X with compact support. We shall use freely terminology and results in the first two chapters of [**12**].

§1. Three properties of a variety.

Let X be an algebraic variety defined over $\boldsymbol{Q}$. There is a finite set S such that for $p \notin S$ the variety $X^{(p)}$ over $\boldsymbol{F}_p$ obtained by the reduction modulo p has at least one rational point over $\boldsymbol{F}_p$ ([7]). We shall say that X is of type (C_1) if it has the following property:

(C_1) *the product* $\prod_{p \notin S}([X^{(p)}_{F_p}]/p^{\dim X})$ *is absolutely convergent.*

Clearly, this property is independent of the choice of S.