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Application of the theory of the group of classes of projective modules to the existence problem of independent parameters of invariant

To celebrate Professor Iyanaga's 60th birthday

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1. Introduction

Let k be a field and let $K=k(x_1, \cdots, x_n)$ be a purely transcendental extension field over k, obtained by adjunction of n elements $x_i (i=1, \cdots, n)$[1] which are mutually independent over k. Let μ denote the automorphism of K/k such that

$$(1) \qquad \mu(x_1)=x_2, \qquad \mu(x_2)=x_3, \qquad \cdots, \qquad \mu(x_n)=x_1 .$$

Let G be the automorphism group of K generated by μ and L the subfield of K consisting of all the elements which are kept elementwise invariant by G. G is a cyclic group of order n, $[K:L]=n$, and K/L is a separable Galois extension, having G as its Galois group. Hence L/k is a finite regular extension of dimension n. Then the following is a classical problem:

PROBLEM. Is L/k a purely transcendental extension?

In this paper we deal only with the non-modular case of this problem. From now on we assume that n is not divisible by the characteristic of k[2]. When k contains a primitive n-th root of 1, the problem is easy and was solved[3] in the affirmative. The most fundamental case of the problem is that k is the rational number field $\boldsymbol{Q}$ and n is a prime integer p. In case of $k=\boldsymbol{Q}$ and $n=p$ the problem has been solved only for $p=2, 3, 5$, and 7[4]. The author proved the pure transcendency of $L/\boldsymbol{Q}$ in cases $p=3, 5$, and 7 as follows (cf. [**3**]). Let T be the p-th cyclomic field and H the Galois group of $T/\boldsymbol{Q}$. Let γ

1) In this paper, we use i and j as index variables. If 0 belongs to the range of the values, we use j exclusively. If not, i.

2) Cf. [1], where the modular case is studied.

3) For example, cf. [**3**], Theorem 1.

4) The first proof for the case $p=3$ is due to E. Nöther. We can see a good bibliography for this classical problem in [**2**].