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Some results on Γ-extensions of algebraic number fields

Dedicated to Professor Iyanaga on his 60th birthday

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Let l denote a prime number which we fix throughout the present paper. Let F_0 be an algebraic number field of finite degree, and let F/F_0 be a Γ-extension over F_0. Namely F/F_0 is a Galois extension whose Galois group is isomorphic to the additive group of l-adic integers. In the following we shall consider a Γ-module $A(K/F)$, attached to F/F_0, which will be defined analogously to the cyclotomic case considered by Iwasawa [8]. After the preliminaries in § 1 we shall give in § 2 a necessary and sufficient condition for the regularity of $A(K/F)$ (as Γ-module) in terms of characters of idèle groups of intermediate fields of F and F_0 (Theorems 1 and 2). The Γ-module $A(K/F)$ is intimately related to l-adic behaviour of global unit groups of algebraic number fields (Theorem 3 in § 2).

Now let in particular the ground field F_0 be an imaginary quadratic extension of the rational number field. In such a case there exist, in a fixed algebraic closure of F_0, two independent Γ-extensions over F_0 (with respect to our fixed prime number l). Under additional conditions on F_0 the regularity of $A(K/F)$ will be obtained in § 3 (Theorem 4 in § 3).

General notations. We denote by l a prime number which we fix throughout the present paper[1]. $\boldsymbol{Z}$ and $\boldsymbol{Q}$ stand for the ring of rational integers and the rational number field, respectively. We denote by $\boldsymbol{Z}_l$ and $\boldsymbol{Q}_l$ the ring of l-adic integers and the l-adic completion of $\boldsymbol{Q}$, respectively. $\boldsymbol{Z}/(d)\boldsymbol{Z}$ means the additive group of integers modulo d, where $d \in \boldsymbol{Z}$.

§ 1. Preliminaries.

1.1.[2] Now let in general E be a field and K/E a Galois extension. Then the Galois group of K/E equipped with the Krull topology will be denoted by $G(K/E)$. Let F be an intermediate field of K and E which is also a Galois extension over E. Then the Galois group $G(F/E)$ is canonically isomorphic to

1) We reserve the notations p, $\mathfrak{p}$, etc. for general prime numbers or prime divisors.
2) Cf. Iwasawa [8], § 1. The purpose of the descriptions in § 1.1 and § 1.2 is to introduce notations.