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Pluricanonical systems on algebraic surfaces of general type

Dedicated to Professor S. Iyanaga on his 60th birthday

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By a minimal non-singular algebraic surface of general type we shall mean a non-singular algebraic surface free from exceptional curves (of the first kind) of which the bigenus P_2 and the Chern number c_1^2 are both positive, where c_1 denote the first Chern class of the surface (see § 3). Let S denote a minimal non-singular algebraic surface of general type defined over the field of complex numbers and let K be a canonical divisor on S. The number of non-singular rational curves E on S satisfying the equation: $KE=0$ is smaller than the second Betti number of S, where KE denotes the intersection multiplicity of K and E. We define $\mathcal{E}$ to be the union of all the non-singular rational curves E with $KE=0$ on S and represent it as a sum: $\mathcal{E}=\sum_{\nu}\mathcal{E}_{\nu}$ of its *connected components* $\mathcal{E}_{\nu}$. Obviously $\mathcal{E}$ may be an empty set. Consider a holomorphic map $\Phi: z\rightarrow\Phi(z)$ of S into a projective n-space $\boldsymbol{P}^n$. We shall say that Φ is *biholomorphic modulo* $\mathcal{E}$ if and only if Φ is biholomorphic on $S-\mathcal{E}$ and $\Phi^{-1}\Phi(z)=\mathcal{E}_{\nu}$ for $z\in\mathcal{E}_{\nu}$. For any positive integer m, we let Φ_{mK} denote the *rational map* of S into $\boldsymbol{P}^n$ defined by the pluri-canonical system $|mK|$, where $n=\dim|mK|$. Note that, if $|mK|$ has no base point, then Φ_{mK} is a holomorphic map. D. Mumford proved that, for every sufficiently large integer m, the pluri-canonical system $|mK|$ has no base point and Φ_{mK} is biholomorphic modulo $\mathcal{E}$ (see Mumford [6]; compare also Zariski [9], Matsusaka and Mumford [5]). His proof is based on results of Zariski [9] and covers the abstract case. On the other hand, it has been shown by Šafarevič [8] that Φ_{9K} is a birational map. The main purpose of this paper is to prove the following theorem:

THEOREM. *For every integer $m\geqq 4$, the pluri-canonical system $|mK|$ has no base point and Φ_{mK} is a holomorphic map. For every integer $m\geqq 6$, the map Φ_{mK} is biholomorphic modulo $\mathcal{E}$.*