J. Math. Soc. Japan
Vol. 20, Nos. 1-2, 1968

Formal functions and formal embeddings

Dedicated to Professor S. Iyanaga

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(Received Sept. 15, 1967)

Introduction.

This work contains two theorems, among others, which determine the field of formal-rational functions, $\hat{K}$, along a closed algebraic set X in a projective space P and in an abelian variety A, respectively. For obvious reasons, we assume that X is connected and has a positive dimension. In the case of ambient variety P, the answer is that $\hat{K}$ is exactly the field of rational functions on P. If A is the ambient variety, $\hat{K}$ coincides with the field of rational functions on a certain abelian scheme A^* over a certain complete local ring R, which is derived from the given pair (X, A). For instance, if X generates A, then R is nothing but the base field of A and A^* is the maximal one, say $\mathrm{Al}(X, A)$, among those étale and proper (hence abelian) extensions of A which are dominated by the albanese variety of X. In the general case, the origin of A being chosen in X with no loss of generality, let A' be the abelian subvariety of A which is generated by X, and $A''=A/A'$. Then R is the completion of the local ring of A'' at the origin, and A^* is the unique etale extension of $A\times_{A''}\mathrm{Spec}(R)$ that induces the covering $\mathrm{Al}(X, A')$ of the closed fibre A'. (There exists a non-canonical isomorphism of A^* with the product $\mathrm{Al}(X, A')\times\mathrm{Spec}(R)$.)

In [1], a general problem was posed about the existence of a certain universal scheme associated with any given formal scheme. The results of this paper readily imply an affirmative and explicit solution to the problem when the given formal scheme is the completion of P (resp. A) along X as above. Namely, in this case, P itself (resp. A^* described above) is the universal solution, i.e., the most dominant (in the sense of an arbitrary small neighborhood of the image of X) scheme of finite type over the ring of formal-regular functions, which in this case, is k (resp. R). Meanwhile, Hartshorne gave an affirmative answer to the same problem, when X is a smooth (or more generally,

* This work has been partially supported by the National Science Foundation under grant No. NSF GP-5177, Senior Foreign Science Fellowship NSF GP-FY67 and the Sloan Foundation.