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A class number associated with the product of an elliptic curve with itself

To Professor Shôkichi Iyanaga for the congraturation of his 60th birthday

By Tsuyoshi HAYASHIDA

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In a previous paper [3] the existence of curves C on the product variety $E\times E'$ of two elliptic curves E and E' with complex multiplication, with the self-intersection number $(C, C)=2$, was proved. $E\times E'$ is then the Jacobian variety of C, C being a theta divisor on $E\times E'$ (Weil [7], Satz 2). The purpose of this paper is to determine explicitly, in a special case $E=E'$, the number of mutually non-isomorphic such curves C of genus 2. More precisely, we shall determine, for a given elliptic curve E with the ring of endomorphisms isomorphic to the principal order of an imaginary quadratic field $\boldsymbol{Q}(\sqrt{-m})$, the number H of isomorphism classes of canonically polarized Jabobian varieties $(E\times E, C)$, C being a theta divisor, as a function of m. In the case $m\equiv 1 \pmod 4$ and $m>1$, for example, we shall obtain the following result:

$$H=\frac{1}{8}\prod_{p}(p-1)\prod_{p}(p+1)+\frac{1}{4}h-2^{t-4},$$

where the first product extends over all prime factors $p\equiv -1 \pmod 4$ of m, and the second over all prime factors $p\equiv 1 \pmod 4$ of m; and h and t are the class number and the number of distinct prime factors of the discriminant of the principal order of $\boldsymbol{Q}(\sqrt{-m})$, respectively. The determination of the number H is reduced to that of the number of classes and the number of "singular" classes of right ideals of certain (non-maximal) orders of a quaternion algebra, and for this purpose Eichler's method ([1] Satz 10) is applicable.

We denote by $\boldsymbol{Q}$ and $\boldsymbol{Z}$ the field of rational numbers and the ring of rational integers, respectively.