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On pseudo groups

Dedicated to Professor Shôkichi Iyanaga on his 60th Birthday

By Richard BRAUER

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Introduction

An investigation of finite groups by means of group characters frequently leads to an array of numbers which looks like a perfectly reasonable table of characters. Actually there may exist groups G with these characters. On the other hand, it may turn out that no such groups exist.

It seems natural to study such arrays. Since they may be considered as generalizations of the system of group characters of finite groups, we will use the term *pseudo groups* for them. We shall see that they share some of the properties of groups. We require relatively little in our definition of pseudo groups in section 1. A stronger axiom is added in section 3. There are further conditions which one might impose. This is discussed briefly in the last section.

1. Definition of pseudo groups

We consider collections

(1.1) $$G=\{K, \Xi, v, E\}$$

where K is a finite set

(1.2) $$K=\{k_1, k_2, \cdots, k_r\},$$

Ξ is a set of complex-valued functions

(1.3) $$\Xi=\{\chi_1, \chi_2, \cdots . \chi_s\}$$

defined on K, v is a complex-valued function defined on K, and E is a set of mappings e_n of K into K, one for each rational integer n.

Each finite group G gives rise to a system (1.1), if the following interpretations are used:

(a) K is the set of conjugate classes of G.

(b) Ξ is the set of irreducible characters of G.

(c) If g is the order of G, then for each conjugate class k_j, $gv(k_j)$ is the number of elements in k_j.