

Supplement to: Holomorphic imbeddings of symmetric domains

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There are some incomplete explanations in p. 274 (§ 2) of my previous paper [1], which I will supplement here. I wish to express my sincere thanks to Professor. I. Satake who has kindly indicated me the incompleteness together with valuable advices.

1. We used a property without proof, which can be formulated as follows:

PROPOSITION. *Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ and $\mathfrak{g}' = \mathfrak{k}' + \mathfrak{p}'$ be Cartan decompositions of Lie algebras of hermitian type without compact factors, $\rho: \mathfrak{g} \rightarrow \mathfrak{g}'$ a homomorphism satisfying the analytic condition (H_2) w.r.t. complex structures H_0 and H'_0 of $(\mathfrak{g}, \mathfrak{k})$ and $(\mathfrak{g}', \mathfrak{k}')$ respectively. Let further $\mathfrak{g} = \sum_{i=1}^e \mathfrak{g}_i$ ($e \geq 2$) be a decomposition of \mathfrak{g} into the direct sum of simple ideals. Then there are regular subalgebras \mathfrak{g}'_j ($1 \leq j \leq e$) of \mathfrak{g}' such that*

i) $\rho(\mathfrak{g}_i) \subset \mathfrak{g}'_i$, and the restriction of ρ to \mathfrak{g}_i satisfies the condition (H_2) w.r.t. the complex structures of $(\mathfrak{g}_i, \mathfrak{k}_i)$ and $(\mathfrak{g}'_i, \mathfrak{k}'_i)$ compatible to those of $(\mathfrak{g}, \mathfrak{k})$ and $(\mathfrak{g}', \mathfrak{k}')$ respectively;

ii) $[\mathfrak{g}'_i, \mathfrak{g}'_j] = 0$ if $i \neq j$.

PROOF. We may assume that \mathfrak{g}' is simple ([1], p. 273). Let H_{0i} be the projection of H_0 to \mathfrak{g}_i , and put $H'_{0i} = \rho(H_{0i})$. Since ρ satisfies (H_2) , we have $H_0 = \sum_{i=1}^e H_{0i}$, $H'_0 = \sum_{i=1}^e H'_{0i}$. Put

$$\mathfrak{v}'_{i+} = \{ \alpha' \in \mathfrak{v}'_+ \mid \alpha'(H'_{0i}) = \sqrt{-1} \}.$$

$$\mathfrak{p}'_i = \mathfrak{g}' \cap \sum_{\alpha' \in \mathfrak{v}'_{i+}} (\mathfrak{g}'_{\alpha'} + \mathfrak{g}'_{-\alpha'}),$$

$$\mathfrak{k}'_i = [\mathfrak{p}'_i, \mathfrak{p}'_i] \quad \text{and} \quad \mathfrak{g}'_i = \mathfrak{k}'_i + \mathfrak{p}'_i,$$

where \mathfrak{v}'_+ denotes the set of all positive non-compact roots, and $\mathfrak{g}'_{\alpha'}$ the root space belonging to α' . Then \mathfrak{g}'_i is a regular subalgebra such that the restriction of ρ to \mathfrak{g}_i defines a homomorphism of \mathfrak{g}_i into \mathfrak{g}'_i satisfying (H_2) w.r.t. H_{0i} and H'_{0i} (cf. [1], Theorem 2). On the other hand, one knows ([2], p. 96) that

$$H'_{0i} = 2\sqrt{-1} \sum_{\alpha' \in \mathfrak{v}'_{i+}} \tilde{\alpha}',$$