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Nonlinear semi-groups in Hilbert space

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In this paper we intend to study the theorem of Hille-Yosida in case of semi-groups of nonlinear contraction operators in Hilbert spaces. Our main results are the following: a nonlinear dissipative operator A in a Hilbert space H generates uniquely a nonlinear semi-group if $(I-A)^{-1}$ is defined on H (Theorem 4), and conversely, the infinitesimal generator A_0 of a nonlinear contraction semi-group in H has an (dissipative) extension A such that $(I-A)^{-1}$ is defined on H and A generates the given nonlinear semi-groups (Theorem 5). Our nonlinear dissipative operator A is in general multi-valued. Hence the set $\{(x, -Ax) \mid x \in D(A)\}$ is monotone in the sense of Minty. Some part of our results (e.g. Theorem 2) is obtained more easily from the theory of monotone operators and of monotone sets by Minty [7]. But we shall make use of the method of semi-groups.

In recent years there appeared many works on nonlinear evolution equations in Hilbert or Banach spaces, for instance see Browder [1], Kato [2], Segal [9] and Sobolevskii [10]. But most of them are concerned with the semilinear case: $\frac{d}{dt} u(t) = A(t)u(t) + f(t, u)$, where $A(t)$ is a linear unbounded operator, and $f(t, \cdot)$ is a nonlinear perturbation.

From the view points of pure theory and its application both, it should be desirable to solve more general nonlinear evolution equations. The author's intension here is to treat the case of not necessarily semilinear (or such) evolution equations.

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1. Nonlinear semi-groups and infinitesimal generators.

Let H be a Hilbert space. A continuous one-parameter semi-group $\{T_t \mid 0 \leqq t < \infty\}$ of nonlinear contraction operators on H is defined by the following conditions:

1) For any fixed $t \geqq 0$, T_t is a continuous (nonlinear) operator defined on H into H.

2) For any fixed $x \in H$, $T_t x$ is strongly continuous in t.