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A remark on theorem A for Stein spaces

By Isao WAKABAYASHI

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1. Concerning Stein spaces the following fundamental theorems of K. Oka—H. Cartan—J. P. Serre are well-known: Theorem A. If $\mathcal{F}$ is a coherent analytic sheaf over a Stein space X with the structure sheaf $\mathcal{O}(X)$, then $\Gamma(X, \mathcal{F})$ generates $\mathcal{F}_x$ for every $x \in X$. Theorem B. In the same case. $\mathrm{H}^q(X, \mathcal{F})=0$ for any $q \geqq 1$.

It is known further that the validity of the latter is sufficient for X to be a Stein space. Namely, a reduced complex space X is a Stein space if $\mathrm{H}^1(X, \mathcal{J})=0$ for any coherent sheaf of ideals $\mathcal{J}$ in $\mathcal{O}(X)$ determined by a zero-dimensional analytic set in X.

In the present note we shall consider the problem if the former is sufficient for X to be a Stein space. Concerning a domain X in $\boldsymbol{C}^n$, Cartan ([1] p. 57) made a remark that, if a certain condition similar to theorem A is satisfied, then X would perhaps be a domain of holomorphy.

Our result is the following:

THEOREM. *Let $(X, \mathcal{O}(X))$ be an n-dimensional reduced connected normal complex space. Suppose it satisfies the following condition* (A): *For any coherent sheaf of ideals $\mathcal{J}$ in $\mathcal{O}(X)$ determined by a zero-dimensional analytic set in X, $\Gamma(X, \mathcal{J})$ generates $\mathcal{J}_x$ as an $\mathcal{O}(X)_x$-module at each point $x \in X$. Then X is* K-*complete and identical with its Kerner's* K-*hull* [4]. *If, in addition, $\Gamma(X, \mathcal{O}(X))$ is isomorphic as a $\boldsymbol{C}$-algebra to $\Gamma(X', \mathcal{O}(X'))$ of an n-dimensional reduced Stein space $(X', \mathcal{O}(X'))$, then X is a Stein space.*

For example, if an unramified covering manifold over a Stein manifold satisfies the condition (A), then it is a Stein manifold. (In general a K-hull of a normal complex space is not necessarily a Stein space [2].)

2. In the following a complex space should be understood to be reduced. A complex space $(X, \mathcal{O}(X))$ is said to be K-complete if, for each point $x \in X$, there exists a holomorphic mapping τ from X to a complex affine space $\boldsymbol{C}^{m_x}$ which is non-degenerate at x, i.e., x is an isolated point of $\tau^{-1}(\tau(x))$. We call a complex space $(X, \mathcal{O}(X))$ a Stein space if it is K-complete and holomorphically convex.

Let $\mathfrak{K}^n$ be a category whose objects are n-dimensional K-complete connected