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Reduction of logics to the primitive logic

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Introduction

Main conclusion of my work [2] has been the following: *Any logic belonging to* **J**-*series* (*the intuitionistic logic* **LJ**, *the minimal logic* **LM**, *and the positive logic* **LP**, *each without assuming Peirce's rule*) *or to* **K**-*series* (*the classical logic* **LK**, *the minimal logic* **LN**, *and the positive logic* **LQ** *which are stronger than* **LJ**, **LM** *and* **LP** *by Peirce's rule, respectively*) *can be faithfully interpreted in the primitive logic* **LO** (*the sub-logic of the intuitionistic logic* **LJ** *having the logical constants, implication and universal quantification, only*). I call here any logic **L** a *sub-logic* of another logic **L*** if and only if every logical constant of **L** is a logical constant of **L*** and every proposition expressible in terms of the logical constant of **L** is provable in **L** if and only if it is provable in **L***.

Faithful interpretation of the intuitionistic logic **LJ** and the classical logic **LK** in the primitive logic **LO** can be realized by $\mathfrak{R}$-*transform* $\mathfrak{A}^{[\mathfrak{R}]}$ of any proposition $\mathfrak{A}$ with respect to an n-ary relation $\mathfrak{R}$. $\mathfrak{A}^{[\mathfrak{R}]}$ can be defined recursively as follows (ξ stands for a sequence of n distinct variables, none of them is assumed to occur free in $\mathfrak{F}$ and $\mathfrak{G}$):

$\mathfrak{F}^{[\mathfrak{R}]} \rightleftarrows (\xi)((\mathfrak{F}\to\mathfrak{R}(\xi))\to\mathfrak{R}(\xi))$ for any elementary formula $\mathfrak{F}$,

$$(\mathfrak{F}\to\mathfrak{G})^{[\mathfrak{R}]} \rightleftarrows (\mathfrak{F}^{[\mathfrak{R}]}\to\mathfrak{G}^{[\mathfrak{R}]}),$$

$$((t)\mathfrak{F})^{[\mathfrak{R}]} \rightleftarrows (t)\mathfrak{F}^{[\mathfrak{R}]},$$

$$(\mathfrak{F}\wedge\mathfrak{G})^{[\mathfrak{R}]} \rightleftarrows (\xi)((\mathfrak{F}^{[\mathfrak{R}]}\to(\mathfrak{G}^{[\mathfrak{R}]}\to\mathfrak{R}(\xi)))\to\mathfrak{R}(\xi)),$$

$$(\mathfrak{F}\vee\mathfrak{G})^{[\mathfrak{R}]} \rightleftarrows (\xi)((\mathfrak{F}^{[\mathfrak{R}]}\to\mathfrak{R}(\xi))\to((\mathfrak{G}^{[\mathfrak{R}]}\to\mathfrak{R}(\xi))\to\mathfrak{R}(\xi))),$$

$$((\exists t)\mathfrak{F})^{[\mathfrak{R}]} \rightleftarrows (\xi)((t)(\mathfrak{F}^{[\mathfrak{R}]}\to\mathfrak{R}(\xi))\to\mathfrak{R}(\xi)),$$

$$(\to\mathfrak{F})^{[\mathfrak{R}]} \equiv \mathfrak{F}^{[\mathfrak{R}]}\to(\xi)\mathfrak{R}(\xi).$$

Now, we can prove the following theorem: $\mathfrak{A}$ *is provable in* **LJ** *if and only if* $\mathfrak{A}^{[R]}$ *is provable in* **LO**, *assuming that* R *is an* n-*ary relation symbol having no occurrence in* $\mathfrak{A}$ *for some* n ($n\geqq 1$). $\mathfrak{A}$ *is provable in* **LK** *if and only if* $\mathfrak{A}^{[R]}$ *is provable in* **LO**, *assuming that* R *is a* 0-*ary relation symbol i.e. proposition symbol having no occurrence in* $\mathfrak{A}$.