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Conformal transformations in complete product Riemannian manifolds

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This paper is a direct continuation of a previous one [7], which we shall refer to for terminologies and notations. In the previous paper, one of the present authors determined the structure of complete Riemannian manifolds admitting a concircular scalar field, and furthermore the structure of complete product Riemannian manifolds admitting a nonisometric conformal vector field under an assumption relative to dimension of manifolds. Here and hereafter we say a vector field to be *isometric* or *conformal* if it generates a one-parameter group of isometric or conformal transformations, respectively. A vector field is said to be *complete* if it generates a global one-parameter group of transformations.

After preliminaries are stated in § 1, we shall study in § 2 the structure of manifolds, named pseudo-hyperbolic spaces in [7], in more details. In § 3, the expression of a concircular scalar field in a space form will be obtained. In § 4, we shall consider complete product Riemannian manifolds admitting a conformal vector field and obtain the equations satisfied by the associated scalar field in a simpler way than that in [7]. As a consequence, the structure of manifolds having such properties is determined without assumption relative to dimension. The purpose of § 5 and of the present paper is to prove the following

MAIN THEOREM. *If a complete reducible Riemannian manifold admits a complete nonisometric conformal vector field, then the manifold is locally Euclidean and the vector field is homothetic.*

This is a generalization of Tanaka's theorem [5] for manifolds with parallel Ricci tensor and of Tachibana's [4] for compact manifolds. Our method of proof is elementary and different from theirs.

§ 1. Preliminaries.

In this paper we shall always deal with connected Riemannian manifolds with positive definite metric, and suppose that manifolds and quantities are differentiable of class C^∞.