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Totally umbilical submanifolds of a Kaehlerian manifold

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Introduction. It is well known fact that an $(n-1)$-dimensional totally umbilical submanifold with non-zero mean curvature of an n-dimensional Euclidean space is isometric with a sphere.

In the previous paper [3], making use of Obata's theorem, the author proved that an $(n-1)$-dimensional complete, simply connected totally umbilical submanifold with non-zero constant mean curvature of an n-dimensional locally product Riemannian manifold is isometric with a sphere.

Now it is natural to try to solve the similar problem in another Riemannian manifold.

On the other hand there are many papers studying submanifolds of an almost complex manifold. However, most of them deals with an invariant submanifold with respect to the almost complex structure. So, this submanifold is necessarily a minimal submanifold. Thus it is expected to study another submanifold of an almost complex manifold.

In this paper, from the above two points of view, the author discusses a totally umbilical submanifold of a Kaehlerian manifold and prove that under some conditions, a $2n$-dimensional totally umbilical submanifold of a $(2n+2)$-dimensional Kaehlerian manifold is isometric with a sphere of $(2n+1)$-dimensional Euclidean space. To prove this we find a function satisfying a certain differential equation. The discovery of such a function enables us to use a famous theorem about an infinitesimal concircular transformation. Thus we can prove the above mentioned theorem completely.

In § 1 we state general properties of $2n$-dimensional submanifold of a $(2n+2)$-dimensional Kaehlerian manifold and in § 2 we give preliminaries of the theory of an infinitesimal concircular transformation.

Finally in § 3 we prove the above theorem under the preparation of some properties of the second fundamental tensor.

§ 1. Submanifolds of a Kaehlerian manifold.

An almost complex manifold $\tilde{M}$ is a differentiable manifold on which there

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