J. Math. Soc. Japan
Vol. 19, No. 3, 1967

Groups with a certain type of Sylow 2-subgroups[1)]

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(Received Nov. 28, 1966)

1. The purpose of this paper is to prove the following theorem:

THEOREM. *Let G be a finite group. If a Sylow 2-group S of G has the following form*

$$S=A\times B$$

where A is a non trivial cyclic 2-group and B is a 2-group with a cyclic subgroup of index 2. Then one of the following possibilities holds:

(a) *S is an elementary abelian 2-group of order 4 or 8.*

(b) *The index $[G:G']$ is even, where G' is the commutator subgroup of G.*

(c) *The group $G/0_{2'}(G)$ has a normal 2-subgroup, where $0_{2'}(G)$ is the maximal normal subgroup of G of odd order.*

In particular, a 2-group S satisfying the assumption of the theorem can be a Sylow 2-subgroup of a simple group only when S is an elementary abelian group of order 4 or 8. Using the argument in proving our theorem, we shall get next proposition.

PROPOSITION. *Let G be a finite group and τ a central involution of a Sylow 2-subgroup of G. If the centralizer of τ $C_G(\tau)$ is isomorphic to the group $\langle\tau\rangle\times PSL(2,q)$ where $q\geqq 5$, then one of the following possibilities holds:*

(a) *S is an elementary abelian 2-group.*

(b) *The factor group $G/0_{2'}(G)$ is isomorphic to the group $\langle\tau\rangle\times PSL(2,q)$. In particular the index $[G:G']=2$.*

This proposition generalizes more or less the following theorem of Z. Janko and J. G. Thompson [5]:

THEOREM. *Let G be a finite group with the following properties:*

(i) *2-Sylow subgroups are abelian,*

(ii) *the index $[G:G']$ is odd,*

(iii) *G has an element τ of order 2 such that*

$$C_G(\tau)=\langle\tau\rangle\times PSL(2,q),\ \textit{where}\ q>5\,.$$

Then G is a non-abelian simple group with $q=3^{2n+1}$ $(n\geqq 1)$.

NOTATION. All the groups considered are finite.

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1) The author thanks to Dr. T. Kondo for pointing out a gap in an original proof of the Theorem.