

Holomorphic imbeddings of symmetric domains

By Shin-ichiro IHARA

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The purpose of this paper is to determine all (equivariant) holomorphic imbeddings of a symmetric domain D into another symmetric domain D' ; a part of results has been announced in [3] without proofs.

In the case D' is of type $(III)_p$ or of type $(I)_{p,q}$, this problem was solved completely (and partially in the case D' is of type $(II)_p$) by Satake in his paper [4]. Our methods are similar to those adopted in [4], but depend further on general properties of Lie algebras. Our results are essentially applicable to any cases.

Let \mathfrak{g} and \mathfrak{g}' be the Lie algebras of the groups of all analytic automorphisms of D and D' respectively. Then the problem is equivalent to that of finding all monomorphisms of \mathfrak{g} into \mathfrak{g}' satisfying a condition called (H_1) in [4]. Therefore we shall consider a slightly generalized problem as was done in [4], that is to determine all *homomorphisms* of a Lie algebra of *hermitian type* into another satisfying (H_1) . A more precise exposition of our problem will be given in § 1.

We shall make reductions of the problem in § 2. Namely, if we find all *regular subalgebras* (see 2.3) of \mathfrak{g}' , and if we determine all pairs (\mathfrak{g}, ρ) of a Lie algebra of hermitian type and a homomorphism of \mathfrak{g} into a regular subalgebra \mathfrak{g}'_ρ of \mathfrak{g}' satisfying a certain condition (H_2) stronger than (H_1) , we shall get all solutions; moreover, we shall be able to assume that both \mathfrak{g} and \mathfrak{g}' are simple. The determination of regular subalgebras of each non-compact simple Lie algebra of hermitian type will be done in § 4. For a simple Lie algebra \mathfrak{g}' of type $(I)_{p,q}$, $(II)_p$, or $(III)_p$, all pairs (\mathfrak{g}, ρ) of a non-compact simple Lie algebra and a homomorphism into \mathfrak{g}' satisfying (H_2) can be determined by combining the results tabulated in [4] and our results in § 4; for the remaining cases $\mathfrak{g}' = (IV)_p$, $(EIII)$, or $(EVII)$, they are determined in § 5. Our § 3 is devoted to the preparations to § 5. In the Appendix, we shall refer to some results, supplementary to those given by Satake in [5], about the correspondence of boundary components by holomorphic imbeddings of symmetric domains.

Throughout this paper, the usual symbols $(I)_p$, $(II)_p$, $(III)_p$, $(IV)_p$, $(EIII)$, and $(EVII)$ for the irreducible symmetric domains will be also used to denote