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Hypersurface with parallel Ricci tensor in a space of constant holomorphic sectional curvature

By Tsunero TAKAHASHI*)

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Introduction. It is well known that the Ricci tensor of a symmetric space is parallel, that is, the covariant derivative of the Ricci tensor vanishes, which is also true in an Einstein space if the dimension of the space is greater than 2. But, in general, the converse is not always true.

The purpose of this paper is to prove the equivalence of the following statements for a complex hypersurface in a space of constant holomorphic sectional curvature: 1) *The hypersurface is a locally symmetric space.* 2) *The hypersurface is an Einstein space.* 3) *The Ricci tensor of the hypersurface is parallel.*

Recently B. Smyth has shown in his thesis that the statement (2) above implies (1) and also he has clasified such a hypersurface [**2**].

§1. Formulas for a complex hypersurface.

In this section we shall summarize the fundamental formulas for a complex hypersurface which will be used in §2. All of them are well known and easily proved as in the case of a real hypersurface [**1**]. The indices $A, B, C, \cdots$ take the values $1, 2, \cdots, n+1$ and the indices $i, j, k, \cdots$ take the values $1, 2, \cdots, n$.

Consider a Kähler manifold M' of complex dimension $n+1$ and a complex hypersurface M in M' which is a complex submanifold of M' of complex codimension 1. M is considered as a Kähler manifold by the induced metric from M'. In terms of local complex coordinates $(z^1, \cdots, z^n)$ of M and $(w^1, \cdots, w^{n+1})$ of M', w^A is a holomorphic function of $(z^1, \cdots, z^n)$. If we denote $\partial w^A/\partial z^i$ by B_i^A, the induced metric tensor $g_{\bar{j}i}$ is given by

$$g_{\bar{j}i} = B_{\bar{j}}{}^{\bar{B}} B_i{}^A g'_{\bar{B}A}\,,$$

where $g'_{\bar{B}A}$ is the metric tensor of M' and $B_{\bar{i}}{}^{\bar{A}} = \overline{B_i{}^A}$.

Let N^A be a complex unit normal vector to M, that is, N^A is defined locally and satisfies

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