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Horizontal lifts from a manifold to its cotangent bundle

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§ 1. Introduction.

The concepts of vertical and complete lifts from a differentiable manifold M of class C^∞ to its cotangent bundle ${}^cT(M)$ were introduced in a recent paper, [4]. Vertical lifts of functions, vector fields, 1-forms and tensor fields of type (1, 1) or (1, 2) were defined. The definitions of complete lifts were restricted to vector fields, tensor fields of type (1, 1) and skew-symmetric tensor fields of type (1, 2). In each case, the complete lift of a tensor field has the same type as the original; however vertical lifts do not have this property. In § 2 of the present paper, we summarise the details of the relevant formulae.

In the present paper we introduce another type of "lift" from M to ${}^cT(M)$, which we call the horizontal lift. We apply our definition to vector fields, tensor fields of type (1, 1) and connections in M. As in the previous paper, we obtain from our construction useful information about the relationships between the structures of M and ${}^cT(M)$.

The most significant difference between the constructions in the present paper and the earlier constructions is that we now assume that a symmetric affine connection is given in the manifold M. The definition of horizontal lift depends upon this connection, whereas the definitions of vertical and complete lifts were independent of connections.

§ 2. Notations and preliminary results.

Throughout, M denotes a differentiable manifold of class C^∞ and of dimension n. Its cotangent bundle is denoted by ${}^cT(M)$ and $\pi: {}^cT(M) \to M$ is the projection mapping. We write U for a coordinate neighbourhood in M and $\pi^{-1}(U)$ for the corresponding coordinate neighbourhood in ${}^cT(M)$.

Suffixes A, B, C, D take the values 1 to $2n$. Suffixes $a, b, c, \cdots, h, i, j, \cdots$ take the values 1 to n and $\bar{i} = i+n$, etc. The summation convention for repeated indices is used. Whenever notations such as $(F_B{}^A)$ are used for matrices, the suffix on the left indicates the column and the suffix on the right indicates