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On the equivalence of Gaussian measures

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§ 1. Introduction.

Let P be a Gaussian measure on the function space $(\boldsymbol{R}^T, \mathscr{B})$, where T is an interval and $\mathscr{B}$ is the σ-algebra generated by all cylinder sets. Then the family of w-functions:

$$X(t, w) = \text{the } t\text{-coordinate of } w,\ w \in \boldsymbol{R}^T,\ t \in T,$$

defines a Gaussian process on the probability measure space $(\boldsymbol{R}^T, \mathscr{B}, P)$. Conversely, every Gaussian process on an arbitrary probability measure space has a representation of such type (coordinate representation). In this paper we shall use only the coordinate representation, unless stated otherwise. Thus we have a one-to-one correspondence between Gaussian processes with the time parameter t in T and Gaussian measures on the function space $\boldsymbol{R}^T$. Two Gaussian processes are said to be *equivalent*, if their corresponding Gaussian measures are equivalent, i. e. mutually absolutely continuous.

J. Hajek [**1**] and J. Feldman [**2**] found independently that two Gaussian measures are either equivalent or singular, and Yu. Rozanov [**3**] established a criterion for the equivalence in terms of the linear operator on $L^2(X)$, Hilbert space spanned by $\{X(t, w)\}$ (the precise definition is given in section 2).

D. Varberg [**7**] has established a necessary and sufficient condition for a class of Gaussian processes to be equivalent to the Brownian motion. He treats the '*factorable*' Gaussian processes, the covariance function of which can be written in the form

$$r(t, s) = \int_T R(t, u)R(s, u)du,$$

where T is a finite interval $[0, b]$. Further he gives conditions on the kernel function of the linear transformation acting on the Brownian path.

Lately L. Shepp [**10**] has solved many problems concerning the *B-equivalence* (the equivalence to the Brownian motion $\{B(t, w)\}$) of a Gaussian process. He has given a simple necessary and sufficient condition on the mean and