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Topological covering of $SL(2)$ over a local field

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The purpose of this paper is to show that, if F is a local field different from the field $\boldsymbol{C}$ of complex numbers, and if F contains the m-th roots of unity, then the topological group $SL(2, F)$ has an m-fold, non-trivial, topological covering group which is of a fairly number-theoretical nature. In the sequel, a local field will always mean a completion, by a finite or infinite place, of an algebraic number field of finite degree, and $SL(2, F)$ will mean the topological group of all 2×2 matrices with determinant 1 over a local field F.

We shall obtain a topological covering of $SL(2, F)$ by proving in a elementary way that an expression containing Hilbert's symbols is actually a factor set of $SL(2, F)$.

Hilbert's symbol[1] of degree m of a local field F containing the m-th roots of unity will be denoted by (α, β), where α, β are non-zero numbers of F. In addition to fundamental properties of Hilbert' symbol, the relation

$$(\alpha, \beta)(-\alpha^{-1}\beta, \alpha+\beta)=1 \tag{1}$$

is useful in our arguments. This formula is valid whenever $\alpha\neq 0$, $\beta\neq 0$, and $\alpha+\beta\neq 0$[2].

Now, our result is the following

THEOREM. *Let $F\neq \boldsymbol{C}$ be a local field containing the m-th roots of unity, let $G=SL(2, F)$, and, for an element $\sigma=\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\in G$, put $x(\sigma)=\gamma$ or δ according to $\gamma\neq 0$ or $=0$. Furthermore, for $\sigma, \tau\in G$, put*

$$a(\sigma, \tau)=(x(\sigma), x(\tau))(-x(\sigma)^{-1}x(\tau), x(\sigma\tau)). \tag{2}$$

Then, $a(\sigma, \tau)$ satisfies the factor set relation

$$a(\sigma, \tau)a(\sigma\tau, \rho)=a(\sigma, \tau\rho)a(\tau, \rho) \tag{3}$$

for any $\sigma, \tau, \rho\in G$, and determines an m-fold topological covering group of G.

The proof will be performed in § 2, and the non-triviality of the covering in the theorem will be treated in § 3.

From the form of the factor set (2), it is understood that our theorem has

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1) As for the definition of Hilbert's symbol, see [**1**].

2) [**1**], p. 55, formula (15).