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A study of transformation groups on manifolds

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§ 0. Introduction.

It might be interesting to ask to what extent the topological and algebraic structures of the group $H(M)$ of the homeomorphisms of a manifold M reflect the topological structure of M. In spite of its importance, unfortunately, little has been known about it. Though it seems very difficult to determine the structures of $H(M)$, many conjectures or problems have been set up by several authors.

Among them, our main concern in this paper is a problem raised by A. M. Gleason and R. S. Palais [2], which is given as follows:

"*Is the closure of a homomorphic image of a connected Lie group G into $H(M)$ locally compact?*"

In this connection, the author in a previous paper [4] has shown the following:

i) If G is a connected Lie group with compact center and if the image of the adjoint representation $Ad(G)$ of G is closed in the general linear group $GL(\mathfrak{g})$ of the Lie algebra $\mathfrak{g}$ of G, then any monomorphic image of G into $H(M)$ is closed and locally compact.

ii) Let φ be a monomorphism from G into $H(M)$. If $\varphi(V)$ has the locally compact closure for any closed vector subgroup V of G, then so does $\varphi(G)$.

iii) If M is connected and one dimensional, then any monomorphic image of any vector group has the locally compact closure.

The manifolds treated in i)—iii) are all assumed to satisfy the second countability axiom and the topology for $H(M)$ is of course the compact open topology. Under the same assumptions, the present author has also shown in [6] the following:

iv) Let φ be a homomorphism from a vector group V into $H(M)$ of a connected manifold M. If every orbit $\varphi(V)(x)$, $x \in M$, is homeomorphic to a circle or a point, then $\varphi(V)$ is closed in $H(M)$ and locally compact.

The object of this paper is to obtain the following theorem and example.

THEOREM A. *Let M be a two-dimensional, metric and connected manifold*

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