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Characteristic classes of some higher order tangent bundles of complex projective spaces

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Introduction

Let M be a sufficiently smooth real compact differentiable manifold. In an earlier paper [**2**, Lemma (2.2)], we have computed elements of $KO(M)$ determined by higher order tangent bundles of M and applied them to find bounds for dimensions of odd order non-singular immersions of real projective spaces to real affine spaces. Purposes of the present paper are to express, in a similar manner, a pth order real tangent bundle of a complex projective space by means of symmetric ith power operations in KO-theory, and to compute characteristic classes of the bundle for $p=2, 3$. For some small p and complex projective spaces of small dimension, the pth order tangent bundle is completely determined in the KO-ring of the space. We compute it for $p=2$ and the space of complex dimension 4. As applications, we find bounds for dimensions of some higher order non-singular immersions of complex projective spaces to real affine spaces. One can see several properties of higher order non-singular immersion, in Feldman's work [**3**, II, Theorem 3.2].

Theorem (1.1) represents pth order real tangent bundles of the complex projective space by the operations in KO-theory. Theorems (1.3), (1.4) are computations of their Stiefel-Whitney classes for $p=2, 3$ and their Pontrjagin classes for $p=2$. These results are used in Theorem (1.5), Corollary (1.6) and we obtain necessary and sufficient conditions of second, third order non-singular immersions of complex projective spaces of certain dimensions to real affine spaces. Corollary (1.6) includes Feldman's example for the complex projective plane [**3**, I, Theorem 6.1, (b)][1].

We compute characteristic classes of powers and symmetric powers of certain real vector space bundles in Section 2, which are used, together with Theorem (1.1) concerning with KO-theory, to prove Theorems (1.3), (1.4) in Section 3. Theorem (1.5), Corollary (1.6) and other similar results are proved in the last section.

1) Details of [**3**, I] is stated in [**6**].