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Complex of differential forms

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1. Introduction

We know that for the finitely generated extension fields of the ground field, the complex of differential forms is isomorphic to a universal complex. Therefore, it seems of interest to investigate the universality of the complex of differential forms of a unitary commutative R-algebra A, R being a commutative ring with unit. This paper is an attempt to find conditions under which the two objects—complex of differential forms of A and a univeral complex over A are same. The main theorems of the paper are

(1) If (U, d) is a universal complex over A such that U_1 is a finitely generated projective A-module then (U, d) and $(A(D), \delta)$ are isomorphic, $(A(D), \delta)$ being the complex of differential forms of A.

(2) If A is a finitely generated algebra over a noetherian commutative ring R such that A is a hereditary ring and if U_1 is reflexive then the complex of differential forms of A is universal.

Since for certain algebras, the two complexes—the complex of differential forms and the universal complexes—are isomorphic, it is interesting to see that they differ quite widely in other cases. The algebra considered here is the algebra $K\{x\}$ of formal power series in one indeterminate x over a field K. We have proved that if (V, ∂) is a universal complex over $K\{x\}$ then V_1 cannot be finitely generated free $K\{x\}$-module whereas the $K\{x\}$-module $D_K(K\{x\})$ of K-derivations of $K\{x\}$ is a free module with basis consisting of one element; thus V_1 cannot be isomorphic to the $K\{x\}$-dual $D_K^*(K\{x\})$.

Throughout this paper R will be a commutative ring with unit.

2. Basic definitions

A *complex* over A is a pair (X, d) where X is an anticommutative regularly graded A-algebra [**1**] and $d: X \to X$ is an R-linear mapping such that (i) $dX_n \subseteqq X_{n+1} \forall n \geqq 0$: (ii) $d(xx') = dx \cdot x' + (-1)^n x \cdot dx'$ for all $x \in X_n$ and $x' \in X$ $(n \geqq 0)$: and (iii) $dd = 0$. If (X, d) and (Y, δ) are two complexes over A then a *complex homomorphism* $f: (X, d) \to (Y, \delta)$ is an A-algebra homomorphism from X into