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Transformations of flows

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§ 0. Preliminaries.

Given a topological space (X, σ) with the family of open sets σ, the corresponding topological measurable space is defined to be the pair $(X, \mathcal{B}_\sigma)$, where $\mathcal{B}_\sigma$ is the family of Borel sets. If in particular σ is defined by a metric with distance function d, then we write it $\mathcal{B}_d$ instead of $\mathcal{B}_\sigma$. Furthermore, if the topological measurable space is furnished with a measure ν, the completion of $\mathcal{B}_\sigma$ or $\mathcal{B}_d$ will be denoted by $\mathcal{L}_\sigma$ or $\mathcal{L}_d$ respectively.

In this paper, for basic measure space is taken Lebesgue space $(M, \mathcal{L}, \mu)$, where $\mathcal{L}$ is the underlying σ-algebra, complete under μ, and $\mu(M)=1$[1].

Let (Ω, d) be a complete metric separable space with distance function d, and probability measure P over $\mathcal{B}_d$. Then $(\Omega, \mathcal{L}_d, P)$ is a Lebesgue space. The family of Lebesgue spaces coincides with that of such probability spaces; any Lebesgue space is isomorphic with $[0, 1]$ endowed with an ordinary probability distribution.

A flow[2] on M is a set $\mathfrak{S}=\{M, T_t, \mu\}=\{M, T_t\}$, where $t\in T=(-\infty, \infty)$, T_t is a one-parameter group of automorphisms on M. In the theory of flows, the choice of Lebesgue space for underlying measure space eliminates measure-theoretic complexities. This depends on the separability properties specific to every Lebesgue space.

Now given a flow $\mathfrak{S}=\{M, T_t, \mu\}$, one can carry it over a metric space Ω, getting a flow $\mathfrak{S}'=\{\Omega, S_t, P\}$ which is isomorphic with $\mathfrak{S}$, with the path $\omega_t=T_t\omega$, $\omega\in\Omega$, continuous in (t, ω). Such a method of representing a given flow on a metric space goes back to Ambrose and Kakutani [**2**].

Among those representations a convenient one is that in which the underlying space Ω consists of measurable t-functions, and S_t is the one-parameter family of shifts acting on Ω. For such a flow (Ω, S_t, P), the trajectory $S_t\omega$, $-\infty<t<\infty$, is also continuous in (t, ω). The advantage of this method consists in the fact that the regularity of the paths and metric topology in

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1) For basic properties of the Lebesgue space, see Rohlin [**6**].

2) Rohlin [**7**] presents an excellent exposition of the theory of automorphisms and flows developed up to 1949.