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A characterisation of exponential distribution semi-groups

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§1. Introduction.

The notion of the (exponential) distribution semi-group of operators in a Banach space was defined by Lions [1]. He characterized the infinitesimal generator of an exponential distribution semi-group by proving generalized Hille-Yosida theorem (cf. also Foias [2], Yoshinaga [3], [4] and Peetre [5]).

In this paper we shall show another characterisation of exponential distribution semi-group. By virtue of this characterisation, we shall define and characterize holomorphic exponential distribution semi-groups. Finally we shall prove a regularity property of holomorphic distribution semi-groups.

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§2. Summary for Lions' results.

We use the following notations: t represents a real variable; $\mathcal{D}_0$ is the space of C_0^∞ functions which vanish in $t<0$; $\mathcal{S}$ is the space of rapidly decreasing C^∞ functions; $\mathcal{E}'$ is the space of distributions with compact support. Let E be a Banach space. If x is an element of E, $\|x\|$ is the norm of x. $L(E, E)$ is the Banach space of bounded linear operators in E. δ_τ is the Dirac distribution concentrated at $t=\tau$.

DEFINITION 1. A distribution semi-group (D.S.G. in short) G is an $L(E, E)$-valued distribution such that

(i) the support of G is contained in $[0, \infty)$,

(ii) $G(\varphi * \psi)=G(\varphi)G(\psi)$, for any φ and ψ in $\mathcal{D}_0$,

(iii) if $\varphi \in \mathcal{D}_0$ and $x \in E$, and if $y=G(\varphi)x$, the distribution Gy defined by $Gy(\varphi)=G(\varphi)y$ is almost everywhere equal to a function $u(t)$ which is continuous for $t \geqq 0$, $u(+0)=y$ and $u(t)=0$ for $t<0$,

(iv) the set $\mathcal{R}=\left\{\sum_{i=1}^{m} G(\varphi_i)x_i \mid \varphi_i \in \mathcal{D}_0,\ x_i \in E\right\}$ is dense in E,

(v) if $x \in E$, $G(\varphi)x=0$ for any $\varphi \in \mathcal{D}_0$, then $x=0$.

DEFINITION 2. A D.S.G. G is called an exponential distribution semi-